CMSC 330: Organization of Programming Languages

Examples of REs & Finite Automata
Describing Regular Expressions

a) \(0(0|1)^*0\)
   - All strings beginning and ending in 0
b) \(((\epsilon|0)1^*)^*\)
   - All strings
c) \((0|1)^*0(0|1)(0|1)\)
   - All strings with 0 as third digit from right
Creating Regular Expressions

For all strings of 0’s and 1’s that...

a) Begin in 1
   – \(1(0|1)^*\)

b) End in 1
   – \((0|1)^*1\)

c) Contains 00
   – \((0|1)^*00(0|1)^*\)

d) Do not contain 00
   – \((01|1)^*(\varepsilon|0)\)
Creating NFA

For all strings of 0’s and 1’s that…

a) Begin in 1

b) End in 1

c) Contains 00

d) Do not contain 00

Based on regular expression
Creating DFA

For all strings of 0’s and 1’s that...

a) Begin in 1

b) End in 1

c) Contains 00

d) Do not contain 00

Swap final / non-final states!
For RE \((a \mid b)^*\)

a) Construct NFA

b) Accept \textit{ababbab}

\texttt{7,5,1,2,6,8,7,5,3,4,6,8,7,5,1,2,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8,7,5,3,4,6,8 accept}
For RE \((a \mid b)^*\)

c) Reduce NFA to DFA

- Start = \(\varepsilon\)-closure(7) = \(\{7, 5, 1, 3, 8\}\)
- \(R = \{7, 5, 1, 3, 8\}\)
- \(r \in R = \{7, 5, 1, 3, 8\}\)  // mark DFA state
- move \((\{7, 5, 1, 3, 8\}, a) = \{2\}\)
  - \(e = \varepsilon\)-closure(\(\{2\}\) = \(\{2, 6, 8, 7, 5, 1, 3\}\)  // new DFA state
  - \(R = R \cup \{2, 6, 8, 7, 5, 1, 3\}\)  // add to R
  - \(\delta = \delta \cup (\{7, 5, 1, 3, 8\}, a, \{2, 6, 8, 7, 5, 1, 3\})\)
- move \((\{7, 5, 1, 3, 8\}, b) = \{4\}\)
  - \(e = \varepsilon\)-closure(\(\{4\}\) = \(\{4, 6, 8, 7, 5, 1, 3\}\)  // new DFA state
  - \(R = R \cup \{4, 6, 8, 7, 5, 1, 3\}\)  // add to R
  - \(\delta = \delta \cup (\{7, 5, 1, 3, 8\}, b, \{4, 6, 8, 7, 5, 1, 3\})\)
For RE \((a \mid b)^*\)

- \(R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
- \(r \in R = \{2,6,8,7,5,1,3\}\) // mark DFA state
- move \((\{2,6,8,7,5,1,3\}, a) = \{2\}\)
  - \(e = \varepsilon\)-closure(\(\{2\}\)) = \(\{2,6,8,7,5,1,3\}\) // existing DFA state
  - \(\delta = \delta \cup (\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})\)
- move \((\{2,6,8,7,5,1,3\}, b) = \{4\}\)
  - \(e = \varepsilon\)-closure(\(\{4\}\)) = \(\{4,6,8,7,5,1,3\}\) // existing DFA state
  - \(\delta = \delta \cup (\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})\)
For RE \((a \mid b)^*\)

- \(R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
- \(r \in R = \{4,6,8,7,5,1,3\}\) // mark DFA state
- move \((\{4,6,8,7,5,1,3\}, a) = \{2\}\)
  - \(e = \varepsilon\)-closure(\{2\}) = \{2,6,8,7,5,1,3\}\) // existing DFA state
  - \(\delta = \delta \cup (\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\})\)
- move \((\{4,6,8,7,5,1,3\}, b) = \{4\}\)
  - \(e = \varepsilon\)-closure(\{4\}) = \{4,6,8,7,5,1,3\}\) // existing DFA state
  - \(\delta = \delta \cup (\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\})\)
- \(R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
  - No more unmarked states to process
- \(F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\}\)
  - Since 8 \(\in F_n\)
For RE \((a \mid b)^*\)

- Resulting DFA

\[ \Sigma = \{a, b\} \]
\[ R = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\} \]
\[ r_0 = \{7,5,1,3,8\} \]
\[ F_d = \{\{7,5,1,3,8\}, \{2,6,8,7,5,1,3\}, \{4,6,8,7,5,1,3\}\} \]
\[ \delta = \{\ (\{7,5,1,3,8\}, a, \{2,6,8,7,5,1,3\}\),
\[\ (\{7,5,1,3,8\}, b, \{4,6,8,7,5,1,3\}\),
\[\ (\{2,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}\),
\[\ (\{2,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\}\),
\[\ (\{4,6,8,7,5,1,3\}, a, \{2,6,8,7,5,1,3\}\),
\[\ (\{4,6,8,7,5,1,3\}, b, \{4,6,8,7,5,1,3\}\) \} \} \]
For RE (a | b )*

- NFA to DFA reduction pictorial
For RE \((a \mid b)^*\)

d) Minimize DFA

- **Initial partitions**
  - \(\text{Accept} \rightarrow \{1,2,3\} \rightarrow P1\)
  - \(\text{Reject} \rightarrow \emptyset\)

- **Split partition?**
  - \(\text{move}(1,a) \rightarrow P1\)
  - \(\text{move}(2,a) \rightarrow P1\)
  - \(\text{move}(3,a) \rightarrow P1\)
  - \(\text{move}(1,b) \rightarrow P1\)
  - \(\text{move}(2,b) \rightarrow P1\)
  - \(\text{move}(3,b) \rightarrow P1\)

Not required, minimization done
For RE \((a^* \mid b^*)^*\)

a) Construct NFA

b) Accept ababbab

11,9,3,1,2,4,10,12,11,9,7,5,6,8,10,12,11,9,3,1,2,4,10,12…
For RE \((a^* | b^*)^*\)

c) Reduce NFA to DFA

- Start = \(\varepsilon\)-closure(1)
  = \{11,9,3,1,4,7,5,8,10,12\}
- \(R = \{\{11,9\ldots 12\}\}\)
- \(r \in R = \{11,9\ldots 12\}\}
- move \({11,9\ldots 12}, a\) = 2
  - \(\varepsilon\)-closure({2}) = \{2,11,9\ldots 12\}
- move \({11,9,\ldots, 12}, b\) = 6
  - \(\varepsilon\)-closure({6}) = \{6,11,9\ldots 12\}
- ... 
- NFA to DFA reduction pictorial
For RE \((a^* | b^* )^*\)

d) Minimize DFA
- Initial partitions
  - Accept \(\rightarrow \{1,2,3\} \rightarrow P1\)
  - Reject \(\rightarrow \emptyset\)
- Split partition?
  - \(\text{move}(1,a) \rightarrow P1\)
  - ...
Not required, minimization done

e) Compare 2 minimized DFAs
- Identical up to state names!