CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Trees & Binary Search Trees

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Trees

- Trees are hierarchical data structures
  - One-to-many relationship between elements
- Tree node / element
  - Contains data
  - Referred to by only 1 (parent) node
  - Contains links to any number of (children) nodes
Trees

• Terminology
  • Root ⇒ node with no parent
  • Leaf ⇒ all nodes with no children
  • Interior ⇒ all nodes with children
Trees

- Terminology
  - Sibling: node with same parent
  - Descendent: children nodes & their descendents
  - Subtree: portion of tree that is a tree by itself
    - a node and its descendents
Trees

• **Terminology**
  
  • Level $\Rightarrow$ is a measure of a node’s distance from root
  
  • Definition of level
    
    - If node is the root of the tree, its level is 1
    
    - Else, the node’s level is 1 + its parent’s level
  
  • Height (depth) $\Rightarrow$ max level of any node in tree

Height = 3
Binary Trees

- Binary tree
  - Tree with 0–2 children per node
    - Left & right child / subtree
Tree Traversal

- Often we want to
  - Find all nodes in tree
  - Determine their relationship
- Can do this by
  - Walking through the tree in a prescribed order
  - Visiting the nodes as they are encountered
- Process is called **tree traversal**
Tree Traversal

• Goal
  • Visit every node in binary tree

• Approaches
  • **Breadth first** ⇒ closer nodes first
  • **Depth first**
    • Preorder ⇒ parent, left child, right child
    • Inorder ⇒ left child, **parent**, right child
    • Postorder ⇒ left child, right child, **parent**

NOTE: left visited before right
Tree Traversal Methods

- **Pre-order**
  1. Visit node // first
  2. Recursively visit left subtree
  3. Recursively visit right subtree

- **In-order**
  1. Recursively visit left subtree
  2. Visit node // second
  3. Recursively right subtree

- **Post-order**
  1. Recursively visit left subtree
  2. Recursively visit right subtree
  3. Visit node // last
Tree Traversal Methods

• Breadth-first

BFS(Node n) {

    Queue Q = new Queue();
    Q.enqueue(n);              // insert node into Q
    while ( !Q.empty()) {
        n = Q.dequeue();       // remove next node
        if ( !n.isEmpty()) {
            visit(n);            // visit node
            Q.enqueue(n.Left());  // insert left subtree in Q
            Q.enqueue(n.Right()); // insert right subtree in Q
        }
    }
}
Tree Traversal Examples

• Breadth-first
  • $+ \times / 2 3 8 4$
• Pre-order (prefix)
  • $+ \times 2 3 / 8 4$
• In-order (infix)
  • $2 \times 3 + 8 / 4$
• Post-order (postfix)
  • $2 3 \times 8 4 / +$

Expression tree
Binary Tree Implementation

- **Choice #1:** Using a class to represent a Node
  
  ```java
  Class Node {
      KeyType key;
      Node left, right;  // null if empty
  }
  ```
  
  Node root = null; // Empty Tree

- **Choice #2:** Using a Polymorphic Binary Tree
  
  - We will talk about this implementation later on
Types of Binary Trees

- **Degenerate**
  - Mostly 1 child / node
  - Height = $O(n)$
  - Similar to linear list

- **Balanced**
  - Mostly 2 child / node
  - Height = $O(\log(n))$
  - $2^{\text{Height}} - 1 = n$ (# of nodes)
  - Useful for searches
Binary Search Trees

• Key property
  • Value at node
    • Smaller values in left subtree
    • Larger values in right subtree

• Example
  • $Y > X$
  • $Y < Z$
Binary Search Trees

- Examples

1. Binary search trees
   - Nodes: 5, 10, 2, 25, 45
   - Structure:
     - 10
       - 5
         - 2
         - 25
         - 45
     - 30

2. Non-binary search tree
   - Nodes: 5, 45, 2, 30, 10
   - Structure:
     - 10
       - 5
         - 2
         - 25
       - 45
         - 30
Tree Traversal Examples

- In-order
  - 17, 32, 44, 48, 50, 62, 78, 88

Sorted order!

Binary search tree
Example Binary Searches

- Find (2)

```
2 < 10, left
2 < 5, left
2 = 2, found
```

```
2 < 5, left
2 = 2, found
```
Example Binary Searches

- Find (25)

```
25 > 10, right
25 < 30, left
25 = 25, found
25 > 5, right
25 < 45, left
25 < 30, left
25 > 10, right
25 = 25, found
```
Binary Search Properties

• Time of search
  • Proportional to height of tree
  • Balanced binary tree
    • $O(\log(n))$ time
  • Degenerate tree
    • $O(n)$ time
    • Like searching linked list / unsorted array

• Requires
  • Ability to compare key values
Binary Search Tree Construction

• How to build & maintain binary trees?
  • Insertion
  • Deletion
• Maintain key property (invariant)
  • Smaller values in left subtree
  • Larger values in right subtree
Binary Search Tree – Insertion

• Algorithm
  1. Perform search for value $X$
  2. Search will end at node $Y$ (if $X$ not in tree)
  3. If $X < Y$, insert new leaf $X$ as new left subtree for $Y$
  4. If $X > Y$, insert new leaf $X$ as new right subtree for $Y$

• Observations
  • $O(\log(n))$ operation for balanced tree
  • Insertions may unbalance tree
Example Insertion

- Insert (20)

20 > 10, right
20 < 30, left
20 < 25, left
Insert 20 on left
Binary Search Tree – Deletion

• Algorithm
  1. Perform search for value X
  2. If X is a leaf, delete X
  3. Else   // must delete internal node
      a) Replace with largest value Y on left subtree
         OR smallest value Z on right subtree
      b) Delete replacement value (Y or Z) from subtree

• Observation
  • $O(\log(n))$ operation for balanced tree
  • Deletions may unbalance tree
Example Deletion (Leaf)

• Delete (25)

25 > 10, right
25 < 30, left
25 = 25, delete
Example Deletion (Internal Node)

- Delete (10)

1. Replacing 10 with largest value in left subtree

2. Replacing 5 with largest value in left subtree

3. Deleting leaf
Example Deletion (Internal Node)

- Delete (10)

Replacing 10 with smallest value in right subtree

Deleting leaf

Resulting tree
Building Maps w/ Search Trees

- Binary Search trees often used to implement maps
  - Each non-empty node contains
    - Key
    - Value
    - Left and right child

- Need to be able to compare keys
  - Generic type `<K extends Comparable<K>>`
    - Denotes any type K that can be compared to K’s
BST (Binary Search Tree) Implementation

- Implementing Tree using traditional approach
- Based on the BST definition below let’s see how to implement typical BST Operations (constructor, add, print, find, isEmpty, isFull, size, height, etc.)

```java
public class BinarySearchTree <K extends Comparable<K>, V> {
    private class Node {
        private K key;
        private V data;
        private Node left, right;
        public Node(K key, V data) {
            this.key = key;
            this.data = data;
        }
    }
    private Node root;
}
```

- See code distribution: BinaryTreeCode.zip
BST Testing

• How can we test the correctness of BST Methods?
• What is the best approach?