Sorting

• Goal
  • Arrange elements in predetermined order
    • Based on key for each element
    • Derived from ability to compare two keys by size

• Properties
  • Stable → relative order of equal keys unchanged
    • Stable: 3, 1, 4, 3, 3, 2 → 1, 2, 3, 3, 3, 4
    • Unstable: 3, 1, 4, 3, 3, 2 → 1, 2, 3, 3, 3, 4
  • In-place → uses only constant additional space
  • External → can efficiently sort large # of keys
    • Most algorithms discussed in lecture are internal and based on arrays
Type of Sorting Algorithms

• **Comparison-based and Linear Algorithms**
  • Comparison-based Algorithms $\Rightarrow$ Only uses pairwise key comparisons
  • Linear Algorithms $\Rightarrow$ Uses additional properties of keys

• **Comparison-based**
  • Proven lower bound of $O(n \log(n))$
  • Examples
    • $O(n^2) \Rightarrow$ Bubblesort, Selection sort, Insertion sort
    • $O(n\log(n)) \Rightarrow$ Treesort, Heapsort, **Quicksort**, Mergesort

• **Linear Algorithms**
  • Counting sort
  • Bucket (bin) sort
  • Radix sort
Bubble Sort

• Approach
  • Iteratively sweep through shrinking portions of list
  • Swap element $x$ with its right neighbor if $x$ is larger

• Performance
  • $O(n^2)$ average / worst case
Bubble Sort Example

**Sweep 1**

```
7 2 8 5 4
2 7 8 5 4
2 7 8 5 4
2 7 5 8 4
2 7 5 4 8
```

**Sweep 2**

```
2 7 5 4 8
2 7 5 4 8
2 5 7 4 8
2 5 4 7 8
2 4 5 7 8
```

**Sweep 3**

```
2 5 4 7 8
2 5 4 7 8
2 4 5 7 8
2 4 5 7 8
```

**Sweep 4**

```
2 4 5 7 8
2 4 5 7 8
2 4 5 7 8
```
Bubble Sort Code

```c
void bubbleSort(int[ ] a) {
    int outer, inner;
    for (outer = a.length - 1; outer > 0; outer--)
        for (inner = 0; inner < outer; inner++)
            if (a[inner] > a[inner + 1])
                swap(a, inner, inner+1);
}

void swap(int a[ ], int x, int y) {
    int temp = a[x];
    a[x] = a[y];
    a[y] = temp;
}
```

How can we improve it?

Swap with right neighbor if larger
Selection Sort

• Approach
  1. Iteratively sweep through shrinking portions of list
  2. Select smallest element found in each sweep
  3. Swap smallest element with front of current list

• Performance
  • $O(n^2)$ average / worst case
Selection Sort Code

```c
void selectionSort(int[] a) {
    int outer, inner, min;
    for (outer = 0; outer < a.length - 1; outer++) {
        min = outer;
        for (inner = outer + 1; inner < a.length; inner++) {
            if (a[inner] < a[min]) {
                min = inner;
            }
        }
        swap(a, outer, min);
    }
}
```

Sweep through array
Find smallest element
Swap with smallest element found
Tree Sort

- **Approach**
  - Insert elements in binary search tree
  - List elements using *inorder* traversal

- **Performance**
  - Binary search tree
    - $O( n \log(n) )$ average case
    - $O( n^2 )$ worst case
  - Balanced binary search tree
    - $O( n \log(n) )$ average / worst case

\[
\{ 7, 2, 8, 5, 4 \}
\]
Heap Sort

• **Approach**
  1. Insert elements in heap
  2. Remove smallest element in heap, repeat
  3. List elements in order of removal from heap

• **Performance**
  • $O(n \log(n))$ average / worst case
Quick Sort

• **Approach**
  1. Select pivot value (near median of list)
  2. Partition elements (into 2 lists) using pivot value
  3. Recursively sort both resulting lists
  4. Concatenate resulting lists
  5. For efficiency pivot needs to partition list evenly

• **Performance**
  • O( n log(n) ) average case
  • O( n² ) worst case

• Used by Arrays.sort
• Runs faster than mergesort in most cases
Quick Sort Algorithm

1. If list below size K
   • Sort w/ other algorithm (e.g. insertion sort)

2. Else pick pivot $x$ and partition $S$ into
   • $L$ elements $< x$
   • $E$ elements $= x$
   • $G$ elements $> x$

3. Quicksort $L$ & $G$

4. Concatenate $L$, $E$ & $G$
   • If not sorting in place
Quick Sort Example

Partition & Sort

Result
Quick Sort Code

```c
void quickSort(int[] a, int x, int y) {
    int pivotIndex;
    if ((y - x) > 0) {
        pivotIndex = partitionList(a, x, y);
        quickSort(a, x, pivotIndex - 1);
        quickSort(a, pivotIndex + 1, y);
    }
}

int partitionList(int[] a, int first, int last) {
    ... // partitions list and returns index of pivot
}
```
Quick Sort Code

```c
int partitionList(int a[], int first, int last) {
    int i, pivot, border;

    pivot = a[first];
    border = first;
    for (i = first + 1; i <= last; i++) {
        if (a[i] <= pivot) {
            border++;
            swap(a, border, i);
        }
    }
    swap(a, first, border);
    return border;
}
```
Merge Sort

• Approach
  1. Partition list of elements into 2 lists
  2. Recursively sort both lists
  3. Given 2 sorted lists, **merge** into 1 sorted list
     • Examine head of both lists
     • Move smaller to end of new list

• Performance
  • **O( n log(n) )** average / worst case

• Used by Collections.sort
Merge Example

2 7
4 5 8
2 4 5 7 8

7 4 5 8
2
7 5 8
2 4
7

2 4 5
7 8
2 4 5 7 8
Merge Sort Example

Split

Merge
Merge Sort Code

```c
void mergeSort(int[] a, int x, int y) {
    int mid = (x + y) / 2;
    if (x != y) {
        mergeSort(a, x, mid);
        mergeSort(a, mid + 1, y);
        merge(a, x, y, mid);
    }
}

void merge(int[] a, int x, int y, int mid) {
    ... // merges 2 adjacent sorted lists in array
}
```
Merge Sort Code

```java
void merge(int[] a, int x, int y, int mid) {
    int j, size = y - x + 1, left = x, right = mid + 1;
    int[] tmp = new int[a.length];

    for (j = 0; j < size; j++)
        if (left > mid)
            tmp[j] = a[right++];
        else
            if (right > y || a[left] < a[right])
                tmp[j] = a[left++];
            else
                tmp[j] = a[right++];

    for (j = 0; j < size; j++)
        a[x + j] = tmp[j];
}
```
## Sorting Properties

<table>
<thead>
<tr>
<th>Name</th>
<th>Comparison Sort</th>
<th>Avg Case Complexity</th>
<th>Worst Case Complexity</th>
<th>In Place</th>
<th>Can be Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble</td>
<td>✓</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Selection</td>
<td>✓</td>
<td>$O(n^2)$</td>
<td>$O(n^2)$</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Tree</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n^2)$</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Heap</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quick</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n^2)$</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Merge</td>
<td>✓</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>
Some References

• President and Sorting
  • [http://www.youtube.com/watch?v=k4RRi_ntQc8](http://www.youtube.com/watch?v=k4RRi_ntQc8)

• Sorting Algorithms Comparison
Links

• Selection vs. Quicksort
  • http://jtf.acm.org/demos/classroom/SortDemo.html
• What different sorting algorithms sound like
  • http://www.youtube.com/watch?v=t8g-iYGHpEA
• Sorting Algorithms
  • http://maven.smith.edu/~thiebaut/java(sort)/demo.html