1. (10 points) Say that a given country has bills with values $3 and $4.
   (a) What is the highest (integer) number of dollars that cannot be achieved with a combination of these bills?
   (b) Prove that all values above that can be achieved.

2. (15 points) For all positive integers $n$, prove that
   \[ 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6} \]

3. (15 points) Prove that $n(n^2 + 5)$ is divisible by 6 for each integer $n \geq 1$.

4. (15 points) Prove that $n^2 < 2^n$ for all integers $n \geq 5$.

5. (15 points) For all positive integers $n$, prove that \(1 + 3 + 5 + \cdots + (2n - 1) = n^2\).

6. (15 points) Let $a_0 = 1$ and $a_{n+1} = 3a_n + 2$. Prove that $a_n = 2 \cdot 3^n - 1$ for all positive integers $n$.

7. (15 points) For all positive integers $n$, prove that
   \[ \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \]