1. Prove that $\sqrt{3}$ is irrational using unique factorization.

2. Prove that $\sqrt{2}$ is irrational using unique factorization.

3. Find the unique prime factorizations of 5850 and 9540. Use these factorizations to compute $\gcd(5850, 9540)$.

4. Compute $\gcd(268, 196)$ using Euclid’s algorithm. Show your work.

5. Find integers $a$ and $b$ such that $657a + 781b = 1$.

6. Say that $33x + 8 \equiv 14 \mod 91$. Find $x$. (Give your answer as an integer between 0 and 90.)

7. Find the smallest positive integer $n$ such that $n/2$ is a perfect square, $n/3$ is a perfect cube, and $n/5$ is a perfect fifth power. (Hint: Think about what must be true of the number’s prime factorization. You can write the answer as a product of powers of primes - there is no need to multiply the number out.)

8. Find the lowest positive integer $x$ such that the following equations are true:

\[
\begin{align*}
  x &\equiv 5 \mod 17 \\
  x &\equiv 8 \mod 31
\end{align*}
\]

9. Find the lowest positive integer $x$ such that the following equations are true:

\[
\begin{align*}
  x &\equiv 1 \mod 7 \\
  x &\equiv 4 \mod 9 \\
  x &\equiv 3 \mod 5
\end{align*}
\]