CMSC250: Discrete Structures

Homework 9: Probability and Combinatorics

Due: July 8

For all problems below, you must show your work (or if there is no work to show, explain how you got the answer you got.)

1. Alice, Bob, and 6 other people need to stand in a line. The only limitation is that Alice must stand closer to the front of the line than Bob. How many different orders could the 8 people stand in?

2. A biased coin has a 60% chance of landing heads on any toss. What is the chance that out of 7 tosses, exactly 5 will be heads?

3. When \((x + y + z)^{14}\) is fully expanded, what will be the coefficient of the \(x^2y^4z^8\) term?

4. How many subsets of odd size are there of a set of size \(n\)?

5. How many integers between 1 and 999,999 contain each of the digits 1, 2, and 3 at least once?

6. \(x\) is a positive integer. We know that \(x \leq 2000\) and that \(\text{gcd}(x, 2000) = 1\). How many possible values are there for \(x\)?

7. A florist shop sells 8 different kinds of flowers. A customers wants a bouquet with 5 different flowers in it. (Using the same type of flower more than once is allowed. Two bouquets with the same number of flowers of each type in them are considered the same.) How many different bouquets can be made?

8. Say that \(|A| = 5\) and \(R\) is a relation on \(A\). How many possible relations \(R\) are there?

9. Say that \(|A| = |B| = 8\) and \(f\) is a function from \(A\) to \(B\).
   - How many possible choices are there for the function \(f\)?
   - How many possible choices are there for \(f\) if the function has to be a bijection?