Due at the start of class Wednesday, June 25, 2014.

Problem 1. Consider the sum
\[ \sum_{k=1}^{n} k^3. \]
(a) Use a non-integral method to show that the sum is between \( n^4/20 \) and \( n^4 \).
(b) Use the integral method to find better upper and lower bounds.

Problem 2. Let A,B,C be three elements (in that order).
(a) Draw the decision tree for bubble sort on these three elements.
(b) Are there any redundant comparisons? If so what are they?

Problem 3. Assume you want to sort a list of \( n \) numbers that are in \( k \) groups, so that the smallest \( n/k \) are first, the next smallest \( n/k \) are second, etc.
(a) Give an algorithm for sorting this list. How many comparisons does your algorithm use (in the worst case)? (This is an upper bound.)
(b) Give a decision tree based lower bound on the number of comparisons needed to produce a single sorted list. (You could base a lower bound on the fact that there really are \( k \) independent sorting problems. Do NOT use this lower bound argument.)
(c) Compare your lower and upper bounds.