Formula Sheet

Sets:

$\mathbb{Z}$ – Integers $\quad$ $\mathbb{Z}^+$ – Positive Integers $\quad$ $\mathbb{N}$ – Natural Numbers

$\mathbb{R}$ – Real Numbers $\quad$ $\mathbb{Q}$ – Rational Numbers

$$A \cup B = \{x \mid x \in A \vee x \in B\} \qquad A \cap B = \{x \mid x \in A \wedge x \in B\}$$
$$A \setminus B = \{x \mid x \in A \wedge x \notin B\} \qquad A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$
$$A \subseteq B \iff \forall a \in A, a \in B \qquad A = B \iff A \subseteq B \wedge B \subseteq A$$

Logic:

$$\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x)$$
$$\neg(\exists x \in D, P(x)) \equiv \forall x \in D, \neg P(x)$$
$$\neg(\forall x \in D, \forall y \in E, P(x,y)) \equiv \exists x \in D, \exists y \in E, \neg P(x,y)$$
$$\neg(\forall x \in D, \exists y \in E, P(x,y)) \equiv \exists x \in D, \forall y \in E, \neg P(x,y)$$
$$\neg(\exists x \in D, \forall y \in E, P(x,y)) \equiv \forall x \in D, \exists y \in E, \neg P(x,y)$$
$$\neg(\exists x \in D, \exists y \in E, P(x,y)) \equiv \forall x \in D, \forall y \in E, \neg P(x,y)$$

$$(p \implies q) \equiv (\neg p \vee q) \equiv (\neg q \implies \neg p) \equiv ((p \wedge \neg q) \implies C)$$
$$(p \iff q) \equiv ((p \implies q) \wedge (q \implies p))$$
$$p \equiv \neg p \implies C$$

Equivalence	Name
$p \wedge \mathrm{T} \equiv p$ $p \vee \mathrm{F} \equiv p$	Identity Laws
$p \vee \mathrm{T} \equiv \mathrm{T}$ $p \wedge \mathrm{F} \equiv \mathrm{F}$	Universal Bound Laws
$p \wedge p \equiv p$ $p \vee p \equiv p$	Idempotent Laws
$\neg(\neg p)$	Double Negation Law
$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws

Equivalence	Name
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation Laws

Definitions:

Let $n \in \mathbb{Z}$:

$$n \text{ is even} \iff \exists k \in \mathbb{Z}, n = 2k \qquad n \text{ is odd} \iff \exists k \in \mathbb{Z}, n = 2k+1$$

Let $n \in \mathbb{Z}, n > 1$:

$$n \text{ is prime} \iff (\forall r, s \in \mathbb{Z}^+, n = r \cdot s \implies (r = 1 \vee s = 1)) \qquad n \text{ is composite} \iff \neg(n \text{ is prime})$$

Let $a, b \in \mathbb{Z}, a \neq 0$:

$$b \mid a \iff \exists k \in \mathbb{Z}, a = bk$$

Let $r \in \mathbb{R}$:

$$r \in \mathbb{Q} \iff \exists a, b \in \mathbb{Z}, (r = \frac{a}{b} \wedge b \neq 0)$$

Let $x \in \mathbb{R}$ and $n \in \mathbb{Z}$:

$$\lfloor x \rfloor = n \iff n \leq x < n+1 \qquad \lceil x \rceil = n \iff n-1 < x \leq n$$

Relations and Functions:

Let R be a function on A. We say:

- R is reflexive $\iff \forall x \in A,\ (x, x) \in R$.
- R is irreflexive $\iff \forall x \in A,\ (x, x) \notin R$.
- R is symmetric $\iff \forall x, y \in A,\ (x, y) \in R \implies (y, x) \in R$.
- R is antisymmetric $\iff \forall x, y \in A,\ (x\,R\,y \wedge y\,R\,x) \implies x = y$.
- R is transitive, $\iff \forall x, y, z \in A,\ (x\,R\,y \wedge y\,R\,z) \implies x\,R\,z$.

Let $f : A \to B$:

- f is injective $\iff \forall x, y, \in A, f(x) = f(y) \implies x = y$
- f is surjective $\iff \forall b \in B, \exists a \in A, f(a) = b$
- f is bijective $\iff$ f is injective and f is surjective

Let $f^{-1} : B \to A$, $a \in A$, $b \in B$:

$$f^{-1}(b) = a \iff f(a) = b$$

Let $f : A \to B$, $g : B \to C$, $g \circ f : A \to C$:

$$\forall x \in A, (g \circ f)(x) = g(f(x))$$

Counting:

Let A be a set and $\{A_1, A_2, \dots A_n\}$ be a partition of A:

$$|A| = |A_1| + |A_2| + \cdots + |A_n|$$

$$P(n, r) = \frac{n!}{(n-r)!} \qquad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$