Homework 4

This assignment is due at 12PM on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified.

Each solution must be written independently by yourself - no collaboration is allowed.

In your answers, you may use results that we have proved in lecture, results from discussion sessions, and results from previous homeworks as building blocks for your solutions. We also allow you to assume the results for the parity of sums and products (i.e. you can assume odd + odd = even, even × odd = even, etc.) Everything else you will have to prove on your own!

Remember that your solutions must be typeset in LaTeX. You can find a template for this homework on the course site. Make sure that each problem (by question number) is separated from each other (i.e. you should not have the solutions to Question 1 and Question 2 on the same page).

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1. **[10 pts]** There are $n \geq 9$ TAs for a course.

   (a) In how many ways can you form three disjoint groups of three TAs each?

   (b) In how many ways can you form three disjoint groups, one with two TAs, one with three TAs, and one with four TAs.

2. **[10 pts]** In Charlie's pizzeria, they offer eight different toppings for a pizza. Moreover you are allowed to "double" any topping if you wish. For example, it is possible to get a pizza with: single "Pepperoni" topping, double "Cheddar Cheese" topping, double "Mushroom" topping and a single "Olive" topping. Given this:

   (a) How many different pizzas are possible?

   (b) Trying to save money, Charlie starts to import his pizza dough from Cronenberg World. While cheaper than Earth-made doughs, the structural integrity of dough from Cronenberg World is far worse. In particular, Charlie finds that his pizzas can now only support 14 toppings in total (doubled-up toppings count as 2).

   Given this, how many different pizzas are possible now?

3. **[10 pts]** There are 27 different windows below the deck of the Ruby Perl.

   (a) On a normal day, to keep the air fresh while the crew sleeps, at least one of the windows has to be open at all times. In how many ways can this be done?

(b) Suppose on one day, one of the mateys contracts a terrible disease that causes her to emit a foul odor at irregular times. The crew figure out that in order to keep the quarters below the deck habitable, they now have to keep at least half of the windows open at all times. How many ways can this be done?

4. **[10 pts]** In how many ways can the elements of $\{1, 2, \ldots, 99\}$ be permuted so that the sum of every two consecutive elements is odd?

5. **[10 pts]** There are 24 teams in the Ranji Cricket League. The teams are first divided into two zones – East Zone and West Zone, each of which consists of 12 teams. Then each zone is divided into four divisions of three teams each. Each division has a distinct name. In how many ways can this be done?

6. **[10 pts]** A segment on the a Best Friend game show is called "How Similar Are You?". In this segment, the two participants, called them Ajax and Ben (who are "best friends") try to prove that they have similar tastes.

First, Ben is taken away from the stage (to prevent him from listening in on the answers), and Ajax is shown a list of n different ice cream flavors. For each flavor, he answers whether or not he likes that ice cream flavor.

Second, Ajax is taken away from the stage (for the same reasons), and Ben is shown the same list of ice cream flavors. Again, for each flavor, he answers whether or not he likes the ice cream flavor.

Consider the following questions:

(a) How many ways are there for Ajax and Ben to choose ice cream flavors that they like, such that the flavors of ice cream that Ben likes includes all the flavors that Ajax likes?

(b) How many ways are there for Ajax and Ben to choose ice cream flavors that they like, such that the flavors of ice cream that Ben likes includes all the flavors that Ajax likes and at least one additional flavor?