

CMSC 330 Quiz 5 Spring 2022

Q1. Lambda Calculus

Consider the following lambda expression.

$\lambda a. \lambda b. b \ c \ \lambda c. d \ f \ a$

Note: To represent λ , you may either copy and paste the symbol λ or just type the characters `L` or `\` in your solutions.

Q1.1. Make the parenthesis explicit

Q1.2. Which of the following are free (or unbound) variables? Select all that apply.

- a
- b
- c
- d
- f

Q1.3. Which of the following are valid α -conversions? Select all that apply.

- $\lambda x. \lambda b. b \ c \ \lambda c. d \ f \ x$
- $\lambda w. \lambda b. b \ c \ \lambda c. d \ f \ a$
- $\lambda a. \lambda b. b \ x \ \lambda x. d \ f \ a$
- $\lambda a. \lambda b. b \ c \ \lambda w. d \ f \ a$
- $\lambda a. \lambda y. y \ c \ \lambda c. d \ f \ a$

Q2. Concepts

For each of the questions below, select whether the given statement is true or false.

Q2.1. Reducing lambda expressions with call-by-name and call-by-value always yields the same result, assuming that all expressions terminate.

T/F

Q2.2. Reducing lambda expressions with call-by-name and call-by-value always takes the same number of steps/reductions.

T/F

Q3. Beta Reduction

Reduce the following lambda calculus expression to the β -normal form.

$(\lambda y. \lambda y. y \ y) \ a \ (\lambda y. y) \ b$

Show each step, including any β -reduction or α -conversion. If there is infinite recursion, write "Infinite Recursion".

Notes:

- You must make all parenthesis explicit before reducing the expression.
- You also must perform valid α -conversions to remove all ambiguity/duplicate variables.
- To represent λ , you may either copy and paste the symbol λ or just type the characters `L` or `\` in your solutions.

Q4. Mystery Operator

Suppose we have a mystery lambda expression `mys` such that for any input x, a , we have the following:

`mys x a = x`

Note: To represent λ , you may either copy and paste the symbol λ or just type the characters `L` or `\` in your solutions.

Q4.1. Give a possible lambda expression for `mys`.

Q4.2. Using the expression from **Q4.1**, reduce the following expression to the β -normal form.

`mys (\lambda x. x)`

Show each step, including any β -reduction or α -conversion. If there is infinite recursion, write "Infinite Recursion".