To Fill or not to Fill: The Gas Station Problem *

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Abstract

In this paper we study several routing problems that generalize shortest paths and the Traveling Salesman Problem. We consider a more general model that incorporates the actual cost in terms of gas prices. We have a vehicle with a given tank capacity. We assume that at each vertex gas may be purchased at a certain price. The objective is to find the cheapest route to go from s to t, or the cheapest tour visiting a given set of locations. Surprisingly, the problem of find the cheapest way to go from s to t can be solved in polynomial time and is not NP-complete. For most other versions however, the problem is NP-complete and we develop polynomial time approximation algorithms for these versions.

1 Introduction

Optimization problems related to computing the shortest (or cheapest) tour visiting a set of locations, or that of computing the shortest path between a pair of locations are pervasive in Computer Science and Operations Research. Typically, the measures that we optimize are in terms of "distance" traveled, or time spent (or in some cases, a combination of the two). There are literally thousands of papers dealing with problems related to shortest-path and tour problems.

In this paper, we consider a more general model that incorporates the actual cost in terms of *gas prices*. We have a vehicle with a given tank capacity of U. In fact, we will assume that U is the distance the vehicle may travel on a full tank of gas (this can easily be obtained by taking the product of the tank size and the mileage per gas unit of the vehicle). Moreover, we may assume that we start with some given amount of gas μ ($\leq U$) in the tank. We assume that at each vertex v gas may be purchased at a price of $c(v)$. This price is the cost of gas per mile. For example if gas costs \$3.40 per gallon and the vehicle can travel for 17 miles per gallon, then the cost per mile is 20 cents.

At each gas station we may fill up some amount of gas to "extend" the range of the vehicle by a certain amount. Moreover, since gas prices vary, the cost depends on where we purchase gas from.

In addition to fluctuating gas prices, there is significant variance in the price of gas between gas stations in different areas. For example, in the Washington DC area alone, the variance in gas prices between gas stations in different areas (on the same day) can be by as much as 20%. Due to different state taxes, gas prices in adjacent states also vary. Finally, one may ask: why do we expect such information to be available? In fact, there are a collection of web sites [2, 1] that currently list gas prices in an area specified by zip code. So it is reasonable to assume that information about

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