Bandwidth Allocation for Multiplexed VBR Video Streams with Deterministic Guarantees

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Abstract

We present a new bandwidth allocation scheme for VBR MPEG-coded video. Compared to source-peak-rate allocation, this scheme achieves significant bandwidth gain while supporting stringent, deterministic QoS requirements. Video sources are modeled by time-varying envelops, and bandwidth is allocated according to the peak rate of the *aggregate* of these envelops. The allocated bandwidth depends on the *arrangement* of the multiplexed streams, which is a measure of the synchronization among the GOPs of different streams. For a random arrangement, we derive the the tail probability distribution of the allocated bandwidth, and use to compute the blocking probability. When multiplexing is done at a video server, video streams can be properly scheduled to minimize their bandwidth requirement. Optimal and suboptimal scheduling strategies for video sources are provided.

1 Introduction

Efficient bandwidth allocation for VBR video traffic transported using a CBR network service requires reducing the variability of the bit rate by means of temporal averaging (or smoothing) on a stream-by-stream basis or spatial averaging (or aggregation) via statistical multiplexing (SM). Temporal averaging has been proposed for both real-time and archived video. For real-time video, the jitter requirement limits the achievable gain from smoothing. For stored video, smoothing often requires exact knowledge of the end-to-end network delay to avoid overflowing and underflowing the client's buffer. Spatial averaging by means of SM is typically used in ATM networks to reduce bandwidth by allocating resources on demand. To achieve multiplexing gains, the aggregate input rate at the multiplexer is allowed to temporarily exceed the output rate. Such use of SM results in possible cell queueing and buffer overflow, the amounts of which depend on the model being used to characterize the traffic. Because typical traffic models are stochastic in nature, it is often believed that SM can be used advantageously only when providing statistical guarantees. Contrary to this belief, we show that in the case of VBR MPEG-coded video traffic, SM can be used to improve bandwidth allocation while supporting stringent, deterministic QoS guarantees.

In principle, supporting deterministic guarantees necessitates the use of deterministic traffic models. We use a source model which is similar, to some extent, to the D-BIND model introduced in [2]. According to this model, a source s_i is represented by a traffic envelop, $\bar{b}_i(t)$, that is parameterized by the 5-tuple $\boldsymbol{E}_i = \left(I^{(i)}_{max},\ P^{(i)}_{max},\ B^{(i)}_{max},\ L^{(i)},\ Q^{(i)}\right)$. The first three parameters denote, respectively, the maximum sizes of I, P, and B frames of s_i (frame sizes are given in ATM cells which are evenly distributed over a frame period). $L^{(i)}$ and $Q^{(i)}$ parameterize the GOP pattern of s_i: $L^{(i)}$ is the length of a GOP (in frames) and $Q^{(i)}$ is the *P-to-P* frame distance within a GOP. $\overline{b}_i(t)$ is a piecewise-constant periodic function which alternates between $I_{max}^{(i)}$, $P_{max}^{(i)}$, and $B_{max}^{(i)}$ according to the GOP pattern of s_i. It represents a time-varying upper bound on the bit rate. Using such an envelop, we provide an efficient scheme for bandwidth allocation that guarantees zero loss rate and small, bounded delay.

2 Allocation Scheme

Consider N MPEG streams, $s_1, \ldots, s_N$. Let $\overline{b}_i(t)$ be the traffic envelop for s_i (assume, for now, that $L^{(i)} = L$ for all i). Suppose that the first frame of s_i arrives at the multiplexer at time t_i, with $t_1 \stackrel{\triangle}{=} 0$ so that the first stream is used as a time reference. The lag in frame durations between a GOP of s_1 and the following GOP of s_i is given by $u_i = t_i \bmod L$. The vector $\boldsymbol{u} = (u_2, u_3, \ldots, u_N)$, referred to as the *arrangement*, completely specifies the synchronization structure for the N streams with regard to their GOPs. Denote the traffic envelop for the superposition of the N streams by $\overline{b}_{tot}(t)$, $\overline{b}_{tot}(t) = \sum_i \overline{b}_i(t - u_i)$. Note that $\overline{b}_{tot}(t)$ is periodic with period L. We define the *effective bandwidth*[1] for N multiplexed streams with arrangement $\boldsymbol{u}$ by:

$$C(\boldsymbol{u}, N) \stackrel{\triangle}{=} \frac{1}{N} \max_{t \geq 0} \overline{b}_{tot}(t) = \frac{1}{N} \max_{t \geq 0} \left(\sum_{i=1}^{N} \overline{b}_i(t - u_i) \right) \tag{1}$$

Since $NC(\boldsymbol{u}, N)$ bounds the peak rate of the *aggregate* traffic, allocating bandwidth of $C(\boldsymbol{u}, N)$ per source ensures that the instantaneous aggregate input rate never exceeds the output rate. A small buffer of N cells is needed in case cells from several sources arrive simultaneously. With this buffer, zero loss rate and a maximum delay of $1/C(\boldsymbol{u}, N)$ time units are guaranteed.

When $L^{(i)}$ varies with i, (1) is still valid with $\widetilde{L}$ replacing L, where

$$\widetilde{L} = \textit{least common multiple of} \{L^{(1)}, L^{(2)}, \ldots, L^{(N)}\} \tag{2}$$

Given that $I_{max}^{(i)} > P_{max}^{(i)} > B_{max}^{(i)}$ for all i, it is easy to see that $NC(\boldsymbol{u}, N) < \sum_i I_{max}^{(i)}$ for most values of $\boldsymbol{u}$. One extreme case for which $NC(\boldsymbol{u}, N) = \sum_i I_{max}^{(i)}$ is when $\boldsymbol{u}$ is the zero vector (e.g., when all streams start simultaneously). In practice, $C(\boldsymbol{u}, N)$ must be updated online upon the admittance of a new stream or the termination of an ongoing one.

3 Admission Control and Blocking Probability

We now investigate the impact of $C(\boldsymbol{u}, N)$ reservation on CAC and blocking probability. To obtain tractable results, our treatment is limited to the homogeneous traffic case, in which all MPEG sources have a common traffic envelop, $\overline{b}(t)$, specified by $(I_{max},\ P_{max},\ B_{max},\ L,\ Q)$. For heterogeneous streams with different maximum frame sizes but similar L and Q, a common traffic envelop can be obtained by taking I_{max} as the largest $I_{max}^{(i)}$ over all i, and the same for P_{max} and B_{max}. We consider the case of aligned boundaries where u_i assumes only integer values for all i.

Let W be the total bandwidth at the multiplexing node. Suppose that N streams have already been admitted and are being allocated their effective bandwidth. Hence, the available free capacity is $W - NC(u, N)$. Our goal is to compute the blocking probability for a new connection request that arrives at the node. Clearly, this probability depends on W, N, $\overline{b}(t)$, and $C(u, N)$. We fix the

[1] This term is not to be confused with the one commonly defined in probabilistic frameworks as in [1].

first three variables, and assume that $\boldsymbol{u}$ is random with a multivariate discrete uniform distribution. Since the node cannot anticipate in advance the arrival time of the first frame of the $(N+1)$th stream, the CAC algorithm must be designed assuming a worst-case scenario. Hence, the blocking probability can be generally defined by $\Pr\{I_{max}^{(N+1)} > W - NC(u,N)\}$, which in the homogeneous case reduces to

$$\text{Blocking Probability} = \Pr\left\{C(u,N) > \frac{W - I_{max}}{N}\right\} \stackrel{\triangle}{=} G_N\left(\frac{W - I_{max}}{N}\right) \tag{3}$$

where $G_N(x) \stackrel{\triangle}{=}$ the complementary distribution function for $C(u,N)$.

3.1 Tail Distribution of $C(\boldsymbol{u},N)$

For the homogeneous case with aligned boundaries, $C(u,N)$ can be written in the form:

$$C(u,N) = \frac{n_I I_{max} + n_P P_{max} + (N - n_I - n_P)B_{max}}{N} \tag{4}$$

where n_I and n_P are random variables with probability space $\{0,1,\ldots,N\}$. Therefore, $G_N(x)$ can be obtained from the joint distribution of (n_I, n_P). Unfortunately, complete specification of this distribution depends on the relative values of I_{max}, P_{max}, and B_{max}. It is, however, possible to obtain the the tail of this distribution under the assumption that $I_{max} > P_{max} > B_{max}$. Notice that obtaining $C(u,N)$ in the aligned boundaries case requires only computing $\overline{b}_{tot}(t)$ in the first L slots (i.e., phases 0 to $L-1$). Let $\overline{b}_{tot,i} \stackrel{\triangle}{=} \overline{b}_{tot}(\tau)$ for all $\tau \in (i, i+1)$. A stream s_i is said to be *in* phase k if $u_i = k$, i.e., s_i sends an I frame during phase k. For all $k \in \{0,\ldots,L-1\}$,

$r_k \stackrel{\triangle}{=}$ number of streams in phase k

$z_k \stackrel{\triangle}{=}$ number of streams in phases that differ from k by a nonzero multiple of Q

r_k and z_k give the numbers of streams sending I and P frames, respectively, during phase k. It is easy to see that for any two phases, k and j, with $\mid k-j \mid =$ a multiple of Q, we have $r_k + z_k = r_j + z_j$.

Proposition 1 *Let phase k be such that $r_k = \max_i r_i$. If $r_k > N/2$, then $n_I = r_k$. Moreover, phase k is the only phase for which $C(u,N) = \overline{b}_{tot}(k)/N$.*

The proof is omitted for brevity. An implication of Proposition 1 is that when $r_k > N/2$, the peak bit rate for the superposed streams during phase k is greater than or equal the peak rate during any other phase. Therefore, we can compute the joint probability for (n_I, n_P) by simply computing the joint probability for (r_k, z_k) for $r_k > N/2$. Accordingly,

$$p_{ij} \stackrel{\triangle}{=} \Pr\{n_I = i, n_P = j\} = \sum_{l=0}^{L-1} \Pr\{r_l = i, z_l = j\}, \quad \text{for any } j \in \{0,\ldots,N-i\} \tag{5}$$

Note that when $i > N/2$, the events $[r_l = i]$ and $[r_m = i]$, $l \neq m$, are mutually exclusive. It can be shown that p_{ij} is given by:

$$p_{ij} = \frac{\begin{pmatrix} N \\ i \end{pmatrix} \begin{pmatrix} N-i \\ j \end{pmatrix} (L/Q - 1)^j \, (L - L/Q)^{N-i-j}}{L^{N-1}} \tag{6}$$

Since (6) is valid for $i > N/2$, x must be chosen sufficiently large so that the event $[C(u,N) > x]$ necessarily implies $[n_I > N/2]$. Let $x^* \triangleq \inf\{x : [C(u,N) > x] \Rightarrow [n_I > N/2]\}$. Then, for $x > x^*$

$$G_N(x) = \sum_{\substack{i,j \text{ such that} \\ f_{ij} > x}} p_{ij} \tag{7}$$

where

$$f_{ij} \triangleq \frac{iI_{max} + jP_{max} + (N-i-j)B_{max}}{N} \tag{8}$$

It is easy to see that

$$x^* = \frac{(N/2)I_{max} + (N/2)P_{max}}{N} = \frac{I_{max} + P_{max}}{2} \tag{9}$$

since any value of $C(u,N)$ that is greater than the RHS of (9) implies necessarily that $n_I > N/2$. For $x > x^*$, $G_N(x)$ is obtained from (6), (7), and (8).

3.2 Blocking Probability

We now derive the blocking probability of connection requests when bandwidth is allocated according to $C(\boldsymbol{u},N)$. To facilitate a comparison with source peak-rate based allocation, we let $W = (N-K)I_{max}$, where K is a nonnegative integer and N is the number of ongoing streams. The probability that a batch of j new connection requests, where $j \geq 1$, are rejected given N ongoing connections is:

$$\begin{aligned}
P_K^{(N,j)} &\triangleq \Pr\{j \text{ new requests are rejected}\} \\
&= \Pr\{jI_{max} > W - NC(\boldsymbol{u},N) \;/\; W \geq NC(\boldsymbol{u},N)\} \\
&= \frac{\Pr\left\{C(\boldsymbol{u},N) > \frac{N-K-j}{N}I_{max},\; C(\boldsymbol{u},N) \leq \frac{N-K}{N}I_{max}\right\}}{\Pr\left\{C(\boldsymbol{u},N) \leq \frac{N-K}{N}I_{max}\right\}} \\
&= \frac{G_N\left(\frac{N-K-j}{N}I_{max}\right) - G_N\left(\frac{N-K}{N}I_{max}\right)}{1 - G_N\left(\frac{N-K}{N}I_{max}\right)}
\end{aligned} \tag{10}$$

The fact that N connections are already admitted implies necessarily that $W \geq NC(\boldsymbol{u},N)$, which must be conditioned on when computing the blocking probability. Since $G_N(x)$ is known only for $x > x^*$, we must choose N sufficiently large so that $G_N\left(\frac{N-K-j}{N}I_{max}\right)$ can be computed. Let $P_{max} = \alpha I_{max}$ for some $0 < \alpha < 1$. Let $N^*(j) \triangleq \min\{n : (n-K-j)I_{max}/n > x^*\}$. It is easy to show that $N^*(j) = \left\lceil \frac{2(K+j)}{1-\alpha} \right\rceil$. Thus, $P_K^{(N,j)}$ is given by (10) for $N \geq N^*(j)$.

A special case of interest is when $K = 0$, i.e., W is equal to the sum of source peak rates of ongoing streams. Thus, under source-peak-rate allocation, no additional connections can be admitted. Under $C(\boldsymbol{u},N)$-based allocation, the blocking probability for j new connection requests reduces to:

$$P_0^{(N,j)} = G_N\left(\frac{N-j}{N}I_{max}\right) \tag{11}$$

In practice, the blocking probability is needed for $j = 1$ since requests seldom arrive in batches.

4 Efficient Scheduling at a Video Server

Since $C(\boldsymbol{u}, N)$ is a function of the arrangement $\boldsymbol{u}$, it is natural to seek the *best* arrangement that produces the *minimal* effective bandwidth. Such an arrangement can be used for efficient scheduling of video streams at a server. The minimum effective bandwidth is defined as:

$$C_{min}(N) = C(\boldsymbol{u}^*, N) \stackrel{\triangle}{=} \min_{u \in \mathcal{U}} C(\boldsymbol{u}, N) \tag{12}$$

where $\mathcal{U}$ is the set of all arrangements of N streams, and $\boldsymbol{u}^*$ is a *best* arrangement. Obtaining $C_{min}(N)$ from (12) by exhaustive search is computationally prohibitive for moderate and large N.

4.1 Optimal Scheduling in the Homogeneous Case

When all streams are characterized by the same envelop, with possibly different starting times, it can be shown that a best arrangment and its minimum effective bandwidth are given by:

$$\boldsymbol{u}^* = (\underbrace{0, 1, 2, \ldots, L-1, 0, 1, 2, \ldots, L-1, \ldots,}_{w \text{ times}} 0, 1, 2, \ldots, N - wL - 1) \tag{13}$$

$$C_{min}(N) = C(\boldsymbol{u}^*, N) = \frac{(w+1)I_{max} + (m-w)P_{max} + (N-1-m)B_{max}}{N} \tag{14}$$

where

$$w \stackrel{\triangle}{=} \text{largest nonnegative integer } k \text{ that satisfies } N > kL$$
$$m \stackrel{\triangle}{=} \text{largest nonnegative integer } k \text{ that satisfies } N > kQ$$

4.2 Suboptimal Scheduling in the Heterogeneous Case

When video sources are characterized by different envelops, $\boldsymbol{u}^*$ in (13) is no longer optimal. In fact, it can be shown that the optimal schedule in this case depends on the exact values of the traffic envelops' parameters. Therefore, it is not possible to obtain a general expression for a *best* arrangement of heterogeneous sources. Instead, we give a suboptimal schedule that is shown to converge at an exponential rate to the optimal one as $N \to \infty$ Such scheduling does not have a closed-form expression. A table of N rows and $\widetilde{L}$ columns is maintained at the server. Each row describes the $\widetilde{L}$ values associated with $\overline{b}_i(\tau - u_i)$. The server also maintains a vector $\boldsymbol{V} = [v_1\ v_2 \cdots v_N]$ where v_i is sum of the N traffic envelops during phase i. Given N ongoing streams at the multiplexer, the $(N+1)$th stream is scheduled to start in a phase k, $k \in \{0, \ldots, \widetilde{L}-1\}$, for which the total bit rate of the N ongoing streams is at minimum, i.e.,

$$u_{N+1} = k \qquad \text{where} \qquad v_k = \min_{0 \le j \le \widetilde{L}-1} v_j \tag{15}$$

References

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Biography

Marwan Krunz received the B.S. in Electrical Engineering from Jordan University, Amman, Jordan, and the M.S. and Ph.D. degrees in Electrical Engineering from Michigan State University in 1992 and 1995, respectively. He is currently a postdoctoral research associate at the University of Maryland Institute for Advanced Computer Studies (UMIACS). His research interests include video characterization, scheduling and buffer management in ATM networks, bandwidth allocation, RSVP-based Internet services, and the transport of video over wireless networks.

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