Bikeshare Theory

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Bikeshare

Bikeshare programs allow customers to:

1. borrow bikes from one station
2. ride them to another station and leave them there

Based on riding patterns (going to work in the morning, tending to ride downhill), the number of bicycles per station gets imbalanced, so you need to send trucks to rebalance the bikes.

We worked on various problems related to optimal rebalancing.

Tradeoff problem–single depot

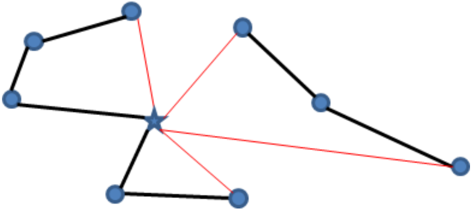
Abstract to generalization of Traveling Salesman Problem:

- Given: central depot, locations to visit, distances. Want to visit locations using k trucks.
- Version 1: Minimize total distance traveled (related to cost for gas)
- Version 2: Minimize maximum tour length (related to time)
- Can't minimize both simultaneously
- Can get within constant factor for both at the same time.
- Found algorithm that gives:
  - $(1.5 + \frac{1}{\beta})$-approximation for total length
  - $(1 + 1.5\beta)$-approximation for maximum tour length

  for any $\beta \geq 1$.

Tradeoff problem–single depot algorithm

1. find 1.5-approximation to the optimal tour (using Christofides) of length L
2. If D is the distance to the furthest client, let $\ell = max(3D, L/k)$
3. Split tour into segments of size $\ell\beta$
4. Connect ends of each segment back



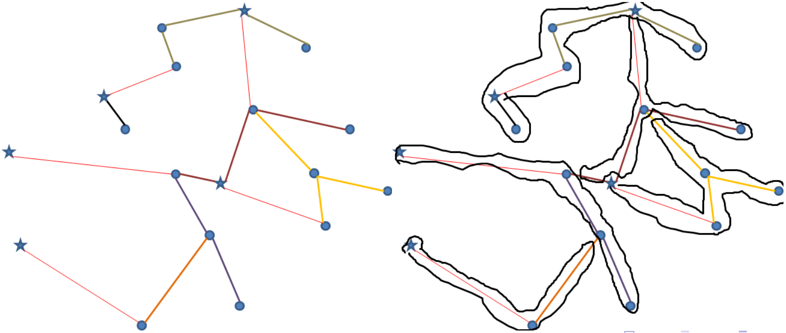
Tradeoff problem–multidepot

Further generalization:

- k depots, each with one truck, must return to same depot
- want to get within a constant factor of minimum for both minimum sum of tour lengths and maximum tour length.
- Can get simultaneously:
  - $(2 + \frac{1}{\beta})$-approximation for the total tour length
  - $$\begin{cases} 1 + 6\beta & \text{for } 1 \leq \beta \leq 1.5 \\ 7 + 2\beta & \text{for } 1.5 \leq \beta \leq 3 \\ 1 + 4\beta & \text{for } 3 \leq \beta \end{cases}$$
    -approximation for maximum tour length for any $\beta \geq 1$.

Tradeoff problem–multidepot algorithm

1. make *GUESS* for the maximum tour length
2. find spanning forest based at the depots
3. split forest into subtrees of size $\beta GUESS$ to $2\beta GUESS$
4. find matching between depots and subtrees that are at most $GUESS/2$ from each other
5. use matching and subtrees to make tours



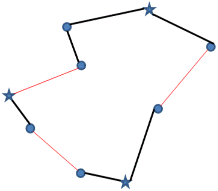
Multidepot TSP

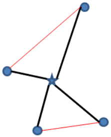
Different problem: minimize sum

- We have k depots, n clients, distances.
- We have trucks go out and return.
- Want to minimize total distance.

What is known:

- NP-hard, but there is 2-approximation. Recent result found 1.5-approximation for constant k (using modified Christofides)
- We tried to eliminate exponential dependence on k
- We tried to apply Christofides directly to multiple depots. Found example showing that no better than 2-approximation





Multidepot capacitated TSP

Different problem:

- We have depots, clients, distances between them.
- We send out trucks, but a truck *can only deliver to c clients* before returning.
- Want to minimize total length.
- Optimal solution NP-hard, but there is a 4-approximation.
- We tried to improve algorithm.