Optimal Diagnosis Procedures for k-out-of-n Structures

MING-FENG CHANG, WEIPING SHI, AND W. KENT FUCHS

Abstract—This paper investigates diagnosis strategies for repairable VLSI and WSI structures based on integrated diagnosis and repair. Knowledge of the repair strategy, the probability of each unit being good, and the expected test time of each unit is used by the diagnosis algorithm to select units for testing. The general problem is described followed by an examination of a specific case. For k-out-of-n structures, we give a complete proof for the optimal diagnosis procedure proposed by Ben-Dov. A compact representation of the optimal diagnosis procedure is described, which requires $O(n^2)$ space and can be generated in $O(n^3)$ time. Simulation results are provided to show the improvement in diagnosis time on off-line repair and on-line repair.

Index Terms—Diagnosis, k-out-of-n, repair, VLSI, wafer probe testing, WSI.

I. INTRODUCTION

Repairable VLSI or WSI structures may contain spare modules (units) for purposes of tolerating manufacturing defects and enhancing yield. In order to tolerate faulty units, it may be necessary to diagnose the system and apply reconfiguration strategies for repair. In the traditional approach, diagnosis and repair have been treated as separate stages: first diagnosis is performed and then the locations of the faulty units are passed to the reconfiguration algorithm [Fig. 1(a)]. An example of this approach is the diagnosis and repair algorithms for repairable memory arrays by Chang, Fuchs, and Patel [1]. However, in some applications the reconfiguration algorithm may need complete information regarding the location of defective units to determine a repair solution. Haddad and Dahbura [2] proposed an on-line repair method which can detect unrepairable random-access memory chips during testing and hence eliminate unnecessary tests. A similar approach was proposed by Huang and Lombardi in their development of a memory repair algorithm which is executed in an on-line fashion with the diagnosis algorithm [3]. The on-line repair approach operates on partial diagnosis information, and therefore can potentially terminate the diagnosis procedure early and provide a repair solution earlier than an off-line repair [Fig. 1(b)].

In what we will call optimal diagnosis and repair, the reconfiguration algorithm not only works on-line with the diagnosis algorithm, but also tells the diagnosis algorithm which unit to test next [Fig. 1(c)]. The reconfiguration and diagnosis algorithms terminate when they determine a repair solution for the structure regardless of the status of the remaining units, or they determine the structure to have been discarded regardless of the status of the remaining units. In addition to exploiting the repair strategy, the diagnosis algorithm can also exploit knowledge about the expected yield of each unit and the expected test time of each unit in the structure.

In this paper, we consider structures whose units are tested individually and sequentially, such as in wafer probe testing [5]. An optimal diagnosis procedure gives the sequence in which units are tested, and minimizes the expected time for determining whether the structure is repairable or unrepairable. The specification of the structure includes the following information: 1) the structure of the system (location and type of each unit, the reconfigurable design),

Fig. 1. Different diagnosis and reconfiguration methods. (a) Traditional off-line repair. (b) On-line repair. (c) Optimal diagnosis and repair.

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2) the probability of each unit being faulty or fault-free, and 3) the expected access and test time for each unit. We believe this paper introduces a new and challenging topic of research concerning integrated diagnosis and repair.

The repairable structure examined in detail in this paper is the $k$-out-of-$n$ structure. Such structures consist of $n$ units, and the structure is functional if at least $k$ units are fault-free. Each unit $u_i$ in the structure has yield $p_i$ and expected test time $t_i$. The optimal diagnosis problem for $k$-out-of-$n$ structures was partially solved by Halpern [6] with the assumption that $t_i = t_j$ for all $i$ and $j$, and also by Salloum and Breuer [7] with the assumption that $t_i/p_i 
eq t_j/p_j$ and $t_i/(1-p_i) 
eq t_j/(1-p_j)$ for all $i$ and $j$. Ben-Dov [8] gave a solution without these restrictions, but his proof (Lemma 2) is incomplete. Other work on this problem includes a branch-and-bound algorithm by Ben-Dov [9], and approximation algorithms by Jedrzejowicz [10].

In this paper, we give a complete proof for the optimal diagnosis procedure proposed by Ben-Dov [8]. We also describe a compact representation of the diagnosis procedure—the compact representation requires only $O(n^2)$ space and can be generated in $O(n^2)$ time.

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In Section II, we briefly describe the problem and notation. Section III gives the optimal diagnosis procedure and proof for $k$-out-of-$n$ structures. Section IV describes the compact representation. Simulation results are presented in Section V.

II. $k$-OUT-OF-$n$ STRUCTURES

A $k$-out-of-$n$ structure consists of a set of $n$ units $N = \{u_1, u_2, \ldots, u_n\}$. Each unit is either faulty or fault-free. The system itself is functional if at least $k$ units are fault-free. For each unit $u_i$, let $p_i$ be the a priori probability that $u_i$ is fault-free, and $t_i$ be the expected time to test $u_i$. For simplicity, let $q_i = 1 - p_i$. A simple 3-out-of-5 structure is shown in Table I. For each unit, its probability of being fault-free (yield) and its expected test time are shown in the table.

A diagnosis procedure can be represented by a binary decision tree. Each nonleaf node specifies a unit to be tested. If the unit is faulty, then the left subtree is taken, otherwise the right subtree is taken. Each leaf node represents the result of the testing, either success or failure. An example diagnosis procedure for the 3-out-of-5 structure is shown in Fig. 2. Clearly, the number of possible diagnosis procedures is exponential in $n$. The objective is to efficiently find the optimal diagnosis procedure which has the minimum expected test time.

A diagnosis procedure for a $k$-out-of-$n$ structure, $P(N,k)$, can be described recursively as

$P(N,k) = \begin{cases} \text{success} & \text{(Test}(u_i) \text{, } P(N - \{u_i\}, k) \text{, } P(N - \{u_i\}, k - 1)) \text{ if } 0 < k \leq n \\ \text{failure} \end{cases}$

where $u_i$ is the unit to be tested first, and if $u_i$ is faulty, then procedure $P(N - \{u_i\}, k)$ is used to test the remaining units, else procedure $P(N - \{u_i\}, k - 1)$ is used to test the remaining units. The diagnosis procedure terminates if $k$ fault-free units have been found or if $n - k + 1$ faulty units have been found. We use $P^*(N,k)$ to represent an optimal diagnosis procedure.

Relabel the units of the system so that

$\frac{f_1}{p_1} \leq \frac{f_2}{p_2} \leq \cdots \leq \frac{f_n}{p_n}$

and let $\pi$ be a permutation so that

$\frac{f_{\pi(1)}}{q_{\pi(1)}} \leq \frac{f_{\pi(2)}}{q_{\pi(2)}} \leq \cdots \leq \frac{f_{\pi(n)}}{q_{\pi(n)}}$.

The optimal diagnosis problem for the highly restricted case where $k = 1$ or $k = n$ was first solved by Butterworth [11]. We briefly repeat his result here for the sake of completeness. In this special case, since all the nonleaf vertices are on a single path, a diagnosis procedure can be simply specified by a linear list of units in $N$.

**Theorem 2.1:** $P^*(N,1) = (u_1, u_2, \ldots, u_n)$ and $P^*(N,n) = (u_{n-k+1}, u_{n-k+2}, \ldots, u_n)$.

III. THE OPTIMAL DIAGNOSIS PROCEDURE

Now we are ready to attack the general problem. Let $U_j = \{u_i | 1 \leq j \leq n \}$ and $V_j = \{u_i | j \leq i \leq n\}$. Intuitively, $U_j$ is the set of units which should be tested first if we want to locate $k$ fault-free units, and $V_{n-k+1}$ is the set of units which should be tested first if we want to locate $n-k+1$ faulty units. The set $U_j \cap V_{n-k+1} \neq \emptyset$, otherwise there would be at least $n+1$ units in $N$ which is a contradiction.

**Theorem 3.1:** The following testing procedure $P(N,k)$ is optimal:

\[ P(N,k) = \begin{cases} \text{success} & \text{if } 0 < k \leq n \\ \text{failure} & \text{if } k = 0 \\ \text{failure} & \text{if } k > n \end{cases} \]

where $u_i$ is any unit in $U_k \cap V_{n-k+1}$.

In order to prove the theorem, we need the following definitions and lemmas.

We associate each unit $u_i \in N$ with an indicator binary variable $x_i$, if $u_i$ is faulty, $x_i = 0$. Otherwise, $x_i = 1$. We call the vector $x = (x_1, x_2, \ldots, x_N)$ an instance, which is a faulty/fault-free combination of units in $N$. The probability that a specific instance $x$ appears is $\prod_i p_i^{x_i} (1-p_i)^{1-x_i}$. Throughout the paper, the bold capital letters are used to represent sets of instances. Let $S_k = \{x | x_i = 1 \text{ and } \sum_{i=1}^N x_i = k \}$, and $F_k = \{x | x_i = 0 \text{ and } \sum_{i=1}^N (1-x_i) = n - k + 1 \}$. It is clear that $S_k$’s and $F_k$’s form a partition of the set of all instances. Notice for $i < k$, $S_i = \emptyset$ and for $i < n-k+1$, $F_i = \emptyset$.

**Lemma 3.1:** For any $x \in S_k$, where $k \leq i \leq n$, the set of units tested by $P(N,k)$ is $U_i$.

**Proof:** By induction on the size of $N$. When $|N| = 1$, it is trivial that $P(\{u_1\}, 1) = \text{Test}(u_1)$, and $U_1 \cap V_1 = \{u_1\}$.

Assume the lemma is true for $|N| < n$. We will prove it is also true for $|N| = n$. For any instance $x \in S_k$, let $u_i$ be the first unit tested by $P(N,k)$. By the definition of $P(N,k)$, $u_i \in U_i \subseteq U_k$.

The remainder of the proof is to show that both $P(N - \{u_i\}, k)$ and $P(N - \{u_i\}, k - 1)$ will test $U_i - \{u_i\}$.

If $u_i$ is faulty, then the $k$th fault-free unit in ascending subscript order, which is $u_i$, is now at position $i - 1$ in $N - \{u_i\}$. Since $N - \{u_i\} \in \mathcal{C}$, from the inductive hypothesis, $P(N - \{u_i\}, k)$ tests the remaining first $i - 1$ units, that is, $U_i - \{u_i\}$.

If $u_i$ is fault-free, then $u_i$ becomes the $(k-1)$th fault-free unit in $N - \{u_i\}$. By inductive hypothesis, $P(N - \{u_i\}, k - 1)$ will test the remaining first $i - 1$ units, that is, $U_i - \{u_i\}$.

\[\Box\]
diagnosis procedure which tests units tested by \( P(N, k) \) as the expected test time when applying the optimal diagnosis procedure, since it is the only procedure.

By a similar argument, we can prove the following.

**Lemma 3.2.** For any \( x \in F_j \), where \( n - k + 1 \leq j \leq n \), the set of units tested by \( P(N, k) \) is \( V_j \).

It is surprising to notice that for any given instance, no matter how we choose \( u_j \in U_j \cap V_{n-k+1} \) when \( |U_j \cap V_{n-k+1}| > 1 \), \( P(N, k) \) always tests the same set of units. We write \( E(P(N, k)) \) as the expected test time when applying \( P(N, k) \) to instance \( x \), over all instances \( x \).

**Proof of Theorem 3.1.*** By induction on the number of units of \( N \). When \( |N| = 1 \), it is trivial that \( P(\{u_i\}) = \text{Test}(u_i) \) is the optimal diagnosis procedure, since it is the only procedure.

Assume \( P(N, k) \) is optimal for all \( N \) of size less than \( n \), we will show that \( P(N, k) \) is optimal for \( |N| = n \). Let \( P(N, k) \) be the diagnosis procedure which tests \( u_i \) first and the remainder diagnosis procedure is optimal. Then

\[
P(N, k) = (\text{Test}(u_i), P^*(N - \{u_i\}, k), P^*(N - \{u_i\}, k - 1))
\]

because \( |N - \{u_i\}| < n \). Since \( E(P^*(N, k)) = \min_{k'} E(P^*(N, k)) \), we will prove \( P(N, k) \) is an optimal diagnosis procedure by showing that

\[
E(P(N, k)) = E(P^*(N, k)) \quad \text{for } u_i \in U_k \cap V_{n-k+1}
\]

\[
E(P(N, k)) \leq E(P^*(N, k)) \quad \text{for } u_i \not\in U_k \cap V_{n-k+1}
\]

If \( u_i \in U_k \cap V_{n-k+1} \), by Lemma 3.1 and Lemma 3.2, \( P^*(N, k) \) and \( P(N, k) \) will test the same set of units for any instance.

1. If \( u_i \not\in U_k \cap V_{n-k+1} \), there are three possible cases:
   1. \( u_i \not\in U_k \cap V_{n-k+1} \)
   2. \( u_i \not\in U_k \cap V_{n-k+1} \)
   3. \( u_i \not\in U_k \cap V_{n-k+1} \)

   We now prove that \( E(P(N, k)) \leq E(P^*(N, k)) \) for case 1. Since \( u_i \not\in V_{n-k+1} \), for any instance in \( F_j \) where \( j > n - k \), \( P(N, k) \) and \( P^*(N, k) \) test the same set of units which is \( V_j \). The fact that \( u_i \) is not in \( U_j \) implies \( j > i \). For any instance in \( S_j \), where \( j > i \), \( P(N, k) \) and \( P^*(N, k) \) test the same set of units, which is \( U_j \). Only for instances in \( \bigcup_{j=1}^{\infty} S_j \), may \( P(N, k) \) and \( P^*(N, k) \) test different sets of units. Therefore, only units in \( U_i \) can be tested. Consider whether each unit \( u_i \in U_i \) is tested or not by \( P(N, k) \) and \( P^*(N, k) \), respectively.

   1.a) If \( j < i \), \( u_i \) is always tested by both procedures.

   1.b) If \( j = i \), \( u_j \) is tested by \( P^*(N, k) \), but is tested by \( P(N, k) \) only for \( x \in S_i \).

   1.c) \( k \leq j < i \). If \( u_i \) is faulty, then \( u_j \) is tested by \( P^*(N, k) \) if and only if \( u_j \) is tested by \( P(N, k) \). If \( u_i \) is fault-free, \( P(N, k) \) needs to locate \( k \) good units in \( U_i \), while \( P^*(N, k) \) needs to locate \( k - 1 \) good units in \( U_{i-1} \). Let \( G(i, g) \) represent the set of instances which have exactly \( g \) good units in \( U_i \). For all the instances in \( \bigcup_{j=1}^{n} G(j - 1, m) \), \( u_i \) is tested by neither diagnosis procedure. For all the instances in \( \bigcup_{j=1}^{n} G(j - 1, m) \), \( u_i \) is tested by both procedures. Only for the instances in \( G(j - 1, k - 1) \), is \( u_j \) tested by \( P(N, k) \) but not by \( P^*(N, k) \).

   Therefore, the difference in the total expected test time is as follows, where \( \text{Prob}(G(j - 1, k - 1)) \) denotes the probability an instance is in \( G(j - 1, k - 1) \), and \( \text{Prob}(S_j) \) denotes the probability an instance is in \( S_j \).

\[
E(P(N, k)) - E(P^*(N, k)) = \sum_{j=1}^{\infty} \text{Prob}(G(j - 1, k - 1))t_j - \sum_{j=1}^{\infty} \text{Prob}(S_j)t_j
\]

Case 1. \( t_j \geq t_j \).

Case 2. \( t_j \geq t_j \).

Case 3. \( t_j \geq t_j \).

IV. COMPACT REPRESENTATION

In this section, we describe how to precompute and save the optimal diagnosis procedure, so that whenever the diagnosis algorithm needs to make a decision, it simply checks the stored procedure in
tree is will show that the binary decision tree can be implicitly represented by the
decision tree requires exponential time and memory space. However, we
tested unit set and test state, then the subtrees rooted by uI and U?
in which a walker can walk from one point to another point on a
rectilinear grid [12].

Definition 4.1: For any vertex v in a binary decision tree, define its
tested unit set TU(v) to be the set of units tested along the path from
the root to v, including v.

Definition 4.2: For any vertex v in a binary decision tree, define its
test state TS(v) to be an ordered pair (i, j), where i and j are the
number of fault-free and faulty units tested along the path from the
root to v, excluding v.

It is obvious that if two vertices v1 and v2 in the tree have the same
tested unit set and test state, then the subtrees rooted by v1 and v2
can be made identical (except that the units tested at the root may be
different). Therefore, we can merge v1 and v2, and let them share the
same diagnosis procedure thereafter. If all the vertices with the same
test state (i, j) can be merged, then let TU((i, j) denote the tested
unit set TU(v) for such that TS(v) = (i, j). If all the vertices with
the same test state can be merged, then the testing procedure can be
simplified to a block-walking representation.

Definition 4.3: Given a k-out-of-n structure N = {u1, u2, . . . , uk}, the block walking representation of its diagnosis
procedure is a 5-tuple (G, S, F, δ1, δ2) where G is the set of intermediate
states, S is the set of success states, F is the set of failure states, and
δ1 and δ2 indicate which unit to test if the last test has succeeded or
failed, respectively. Formally, we have

\[ G = \{(i, j)|0 \leq i \leq k, 1 \leq j \leq n - k\}, \]
\[ S = \{(k, j)|0 \leq j \leq n - k\}, \]
\[ F = \{(i, n - k + 1)|0 \leq i \leq k - 1\}, \]
\[ \delta_1, \delta_2: G \rightarrow N. \]

For simplicity, we write δG(i, j) as δ(i, j), and δF(i, j) as
δF(i, j). To illustrate the meaning of the functions δ1 and δ2, consider an
example δ1(i, j) = u1 and δ2(i, j) = u2. This means that at state
(i, j), we will test unit u1 if the last test succeeded (passed) and we
will test unit u2 if the last test failed. The test before state (0, 0) is
assumed to have succeeded. Fig. 3 is a block-walking representation
of the diagnosis procedure shown in Fig. 2.

Each grid point (i, j) represents a test state (i, j) of the diagnosis
procedure. The diagnosis starts from the grid point (0, 0) and trav-
eses downward. At each point a unit is tested and the diagnosis
moves to the next point depending on the outcome of the test.

It should be pointed out that since the optimal diagnosis pro-
dure is not necessarily unique, there may be some optimal diagnosis
procedures which cannot be represented by the block-walking rep-
resentation. However, the following theorem shows that it is always
possible to describe one optimal procedure using the block-walking
representation, which is indeed what we need.

Theorem 4.1: For any X ⊆ N, define SS(X) to be the unit in X
with the smallest subscript. If we choose u = SS(Uk ∩ Vn-k+1) in
P(N, k), then the diagnosis procedure can be represented by a
block-walking representation.

Proof: This procedure is optimal by Theorem 3.1. The remain-
der of the proof is to show that for any vertices v1 and v2 in the binary
decision tree, TU(v1) = TU(v2) if TS(v1) = TS(v2). Clearly, v1
and v2 must be on the same level of the decision tree. The proof is
by induction on m, the length of the path from the root to v1 and v2.

The base case in which m = 0 is trivial, since only the root has
the test state (0, 0). Also, for the states (i, j) with i = 0 or j = 0,
there exists only one vertex for each state.

Now assume the theorem is true for all vertices in the tree above
level m. In other words, assume all vertices with test state (i, j),
where i + j < m, have the same tested unit set. Replace the top
level m − 1 levels of the tree by a block-walking representation. Consider
an arbitrary test state (i, j) with i + j = m and i, j > 0. Clearly,
there are only two states (i − 1, j) and (i, j − 1) which can possibly
have an edge to state (i, j). By induction hypothesis, (i − 1, j) and
(i, j − 1) are two points in the block-walking representation.

Therefore, there are at most two vertices v1 and v2 with test state
(i, j), where v1 is a son of point (i − 1, j) and v2 is a son of point
(i, j − 1). Therefore,

\[ TU(v_1) = TU((i − 1, j − 1)) \cup \{\delta_2(i − 1, j), \delta_1(v_1)\} \]
\[ TU(v_2) = TU((i − 1, j − 1)) \cup \{\delta_1(i, j − 1), \delta_2(v_2)\}. \]

Hence, TU(v1) = TU(v2) if \{δ1(i − 1, j), δ1(v1)\} = \{δ2(i, j − 1), δ2(v2)\}.

From the following observation,

\[ \delta_1(i − 1, j) = SS(U_{k+1} ∩ V_{n-k+1} − TU((i − 1, j − 1)) \]
\[ \delta_1(v_1) = SS(U_{k+1} ∩ V_{n-k+1} − TU((i − 1, j − 1))) \]
\[ \delta_2(i, j − 1) = SS(U_{k+1} ∩ V_{n-k+1} − TU((i − 1, j − 1)) \]
\[ \delta_2(v_2) = SS(U_{k+1} ∩ V_{n-k+1}). \]

We conclude \{δ1(i − 1, j), δ1(v1)\} = \{δ2(i, j − 1), δ2(v2)\}, be-
cause both sets consist of two units which have the smallest two
subscripts in \{uk, vk\}. Also, for the states (i, j) and (i, j − 1), if
we choose u1 = SS(Uk ∩ Vn-k+1) and u2 = SS(Uk ∩ Vn-k+2),
we can construct the optimal procedure easily. Finally, we only
show how to compute δ1 and δ2.

1: tmp := U1 ∩ Vn-k+1;
\[ \delta(0, 0) := SS(tmp); \]
2: for i := 1 to k − 1
begin
\[ \text{tmp} := \text{tmp} \cup \{u_{(i+1-k+1)}\} \} = \{\delta(0, i − 1)\}; \]
\[ \delta(0, i) := SS(tmp) \]
end;
3: for i := 0 to k − 1
begin
\[ \text{tmp} := V_{n-k+1} \} = \{\delta(0, 0)|0 \leq j \leq i\}; \]
V. SIMULATIONS

Simulations were performed to determine the reduction in the test time by using the optimal diagnosis procedure as compared to other approaches. Three other diagnosis procedures were chosen for comparison:

- **T/P procedure**: the diagnosis procedure always tests the unit with minimum \( t_i/p_i \).
- **T/Q procedure**: the diagnosis procedure always tests the unit with minimum \( t_i/q_i \).

Random procedure: the test sequence is chosen at random.

In Fig. 4, the test time for each unit was set to 1 and the fault-free probability was chosen randomly in a uniform distribution on \((0.3, 0.7)\). Since the \( T/P \) procedure and optimal testing procedure will test the same set of units for a functional instance of the structure, when the yield of the entire structure is high, their performances are very close. On the other hand, when the yield of the structure is low, the performance of the \( T/Q \) procedure is near optimal.

In Fig. 5(a), the test time for each unit was set to 1 and the fault-free probability was chosen as a uniform distribution on \((0.01, 0.99)\). For this case in which the variance of \( p_i \) is large, the improvement of the optimal procedure over the random procedure is more than 17% as shown in the example in Fig. 5(a).

When \( k \) is close to \( n \), the performance of the \( T/P \) procedure is close to that of the optimal diagnosis procedure [Fig. 5(b)], because \( n - k + 1 \) is small (only a small number of faulty units need be located) and it is more cost-effective to initially test units which are more likely to be faulty. Symmetrically, when \( k \) is small, the performance of the \( T/Q \) procedure is near optimal. The expected diagnosis time for two examples with small \( n \), 16, are shown in Fig. 6. The results are similar to the examples with large \( n \).

Let the expected test time for faulty units be \( T_f \) and the test time for fault-free units be \( T_f \). Two examples with \( T_f = 0.5 \) and \( T_f = 0.25 \), respectively, are shown in Fig. 7. Since the test time for a faulty unit is less than that of a fault-free unit, the performance of the \( T/Q \) procedure is close to that of the optimal procedure as \( T_f \) decreases.

VI. CONCLUSION

This paper has described the problem of wafer probe-based optimal diagnosis and repair for structures in which the yield of indi-
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Fig. 6. The expected diagnosis time of two k-out-of-16 structures. (a) A k-out-of-16 system \( (p;\, 0.3 - 0.7) \). (b) A k-out-of-16 system \( (p;\, 0.71 - 0.99) \).

Fig. 7. k-out-of-256 structures with different \( T_f, p;\, (0.3, 0.7) \). (a) \( T_f = 1, T_s = 0.5 \). (b) \( T_f = 1, T_s = 0.25 \).

individual units and the expected time for testing each unit is known. A proof of the optimality of a diagnosis algorithm was given for k-out-of-n structures along with a method of compactly representing an optimal solution. Simulation results illustrate the reduction in diagnosis time achieved by exploiting knowledge of the repair strategy as well as knowledge of the expected yield and test time of each unit in the structure.

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The Full-Use-of-Suitable-Spares (FUSS) Approach to Hardware Reconfiguration for Fault-Tolerant Processor Arrays

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Abstract—A general approach to hardware reconfiguration is proposed for VLSI/WSI fault-tolerant processor arrays. The technique, called FUSS (full use of suitable spares), uses an indicator vector, the

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