Homework 1, MORALLY Due Feb 5

1. (10 points) When is the first midterm (give Date and Time)? When is the second midterm (give Date and Time)? When is the final (give Date and Time)? By when do you have to inform Professor Gasarch that you cannot make the timeslot of the first or second midterm? Of the final?

   SOLUTION TO PROBLEM 1

   MIDTERM ONE: Tue Feb 25, in class.
   MIDTERM TWO: Tue Apr 2, in class.
   FINAL: WED May 22 10:30AM-12:30AM.

   If you can’t make any of these then you must inform Professor Gasarch by Feb 3, 2015.

2. (30 points) Give a Propositional Formula on four variables that has exactly three satisfying assignments. Give the satisfying assignments.

   SOLUTION TO PROBLEM 2

   \((x_1 \land x_2 \land x_3 \land x_4) \lor (\neg(x_1) \land x_2 \land x_3 \land x_4) \lor (\neg(x_1) \land \neg(x_2) \land x_3 \land x_4)\)

   The only satisfying assignments are \((T,T,T,T)\) and \((F,T,T,T)\) and \((F,F,T,T)\).

3. (30 points)
   - Do a truth table for \((p \Rightarrow q) \Rightarrow r\).
   - Do a truth table for \(p \Rightarrow (q \Rightarrow r)\).
   - Are they equivalent? If NOT then state a row where they differ.

   SOLUTION TO PROBLEM 3

   SHORT CUT: Recall that the only way that \(x \Rightarrow y\) is \(F\) is if \(x\) is \(T\) and \(y\) is \(F\).

   The only way \(p \Rightarrow (q \Rightarrow r)\) is \(F\) is if \(p\) is \(T\) and \(q \Rightarrow r\) is \(F\). The latter can only happen if \(q\) is \(T\) and \(r\) is \(F\). Hence the only way \(p \Rightarrow (q \Rightarrow r)\) is \(F\) is if \(p\) is \(T\), \(q\) is \(T\), and \(r\) is \(F\).

   The only way \((p \Rightarrow q) \Rightarrow r\) is \(F\) is if \((p \Rightarrow q)\) is \(T\) and \(r\) is \(F\). \((p \Rightarrow q)\) is \(T\) when \((p,q)\) is either \((T,T)\), \((F,T)\) or \((F,F)\). Hence \((p \Rightarrow q) \Rightarrow r\) is \(F\) when \((p,q,r)\) is either \((T,T,F)\), \((F,T,F)\), \((F,F,F)\).
\[
\begin{array}{ccc|c|c}
 p & q & r & p \Rightarrow (q \Rightarrow r) & (p \Rightarrow q) \Rightarrow r \\
\hline
 T & T & T & T & T \\
 T & T & F & F & F \\
 T & F & T & T & T \\
 T & F & F & T & T \\
 F & T & T & T & T \\
 F & T & F & T & T \\
 F & F & T & T & T \\
 F & F & F & T & F \\
\end{array}
\]

NOT equiv: they differ on the row (F,T,F).

4. (30 points) Show that, for all even \( n \), there exists a formula that is satisfied by exactly \( n \) satisfying assignments. Give the satisfying assignments. (This is NOT by induction. Just give the Formula. You may use DOT DOT DOT though it should be clear what you mean.)

SOLUTION TO PROBLEM 4.

Let \( \phi(i, n) \) be the Boolean formula on \( x_1, \ldots, x_n \) where \( x_1, \ldots, x_i \) are NEGATED but the rest are not. For examples
\[
\phi(0, n) \text{ is } (x_1 \land x_2 \land \cdots \land x_n)
\]
\[
\phi(3, n) \text{ is } (\neg x_1 \land \neg x_2 \land \neg x_3 \land x_4 \land x_5 \land \cdots \land x_n)
\]

Our formulas is
\[
\phi(0, n) \lor \phi(1, n) \lor \cdots \lor \phi(n - 1, n)
\]

The satisfying assignments are
\[
(T, \ldots, T) \ (n T's)
\]
\[
(F, T, \ldots, T) \ (1 F and then n T's)
\]
\[
(F, F, T, \ldots, T) \ (2 F and then n - 1 T's)
\]
\[
\vdots
\]
\[
(F, F, F, \ldots, F, T) \ (n - 1 F and then 1 T's)
\]

There are other options. Perhaps more pleasing would be \( \phi(1, n) \lor \cdots \lor \phi(n, n) \).