Homework 4 MORALLY Due March 07 at 9:00AM
WARNING: THIS HW IS SIX PAGES LONG!!!!!!!!!!!!!!!!!!

1. (0 points but please DO IT) What is your name?

2. (20 points)

   (a) (20 points) Show that, for all primes p, $p^{1/5} \notin \mathsf{Q}$. USE the Unique Factorization.

   (b) (0 points) Try to do it using the MOD method. Discuss if this works or not.

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3. (20 points) Let PRIMES be the set of primes.

   Let $\mathrm{MOD}_{i,j}$ be the set of numbers that are $\equiv i \pmod{j}$.

   Let $\mathrm{PRIMES}_{i,j} = \mathrm{PRIMES} \cap \mathrm{MOD}_{i,j}$.

   For example

   $\mathrm{PRIMES}_{1,4}$ is the set of primes that are $\equiv 1 \pmod{4}$.

   (a) Show that $\mathrm{PRIMES}_{3,4}$ is infinite. (Hint: If $p_1, \ldots, p_L$ are all of the primes $\equiv 3 \pmod{4}$ then look at $4p_1 \cdots p_L - 1$.)

   (b) Show that $\mathrm{PRIMES}_{5,6}$ is infinite.

   (c) Give an infinite sequence $i_1 < j_1 < i_2 < j_2 < \cdots$ such that
   $\mathrm{PRIMES}_{i_1,j_1}$ is EMPTY.
   $\mathrm{PRIMES}_{i_2,j_2}$ is EMPTY.
   $\mathrm{PRIMES}_{i_3,j_3}$ is EMPTY.
   etc.

   (d) Give an infinite sequence $i_1 < j_1 < i_2 < j_2 < \cdots$ such that
   $\mathrm{PRIMES}_{i_1,j_1}$ is FINITE but not EMPTY.
   $\mathrm{PRIMES}_{i_2,j_2}$ is FINITE but not EMPTY.
   $\mathrm{PRIMES}_{i_3,j_3}$ is FINITE but not EMPTY.
   etc.

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4. (30 points- 10 points each) For each of the following sequences find a *simple* function $A(n)$ such that the sequence is $A(1)$, $A(2)$, ... (I am not going to define simple rigorously, but just keep it simple.)

   (a) $10, -17, 24, -31, 38, -45, 52, \ldots$.

   (b) $-1, 1, 5, 13, 29, 61, 125, \ldots$.

   (c) $6, 9, 14, 21, 30, 41, 54, \ldots$.

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5. (25 points) In Grand Fenwick they have two types of coins: one worth 100 cents, and one worth 101 cents.

   (a) (15 points) Show that for all $n \geq 9900$ one can form n with these two types of coins.

   (In Math:

   $$(\forall n \geq 9900)(\exists x, y \in \mathsf{N})[n = 100x + 101y].$$

)

   (b) (10 points) Prove or Disprove: There is NO way to form 9899 cents in Grand Fenwick.

   (In Math:

   $$(\forall x, y \in \mathsf{N})[9899 \neq 100x + 101y].$$

)

   (c) (0 points but you should do it) If you DID NOT KNOW the bound of 9900 how would you find it. HINT: Use Wolfram Alpha.

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6. (Extra Credit) Prove or Disprove: there is a second order sentence that is TRUE of $\mathsf{Q} + \mathsf{Q}$ but false of Q.