Homework 5 MORALLY Due Mar 14 at 9:00AM
WARNING: THIS HW IS FIVE PAGES LONG!!!!!!!!!!!!!!!!!

1. (0 points but please DO IT) What is your name?

2. (20 points- AND if you got the Dup-Spoiler Question wrong on the untimed midterm, but get THIS question right, you will get FULL CREDIT on the that question)

   For this problem you may ASSUME the following

   - For all n, for all $a \ge 2^n$, DUP wins $(\mathsf{N} + \mathsf{N}^*, L_a; n)$.
   - For all n, for all $a \ge 2^n$, DUP wins $(\mathsf{N} + \mathsf{Z} + \mathsf{N}^*, L_a; n)$.
   - For all n, for all $a \ge 2^n$, DUP wins $(\mathsf{N} + \mathsf{Z} + \mathsf{Z} + \mathsf{N}^*, L_a; n)$.
   - For all n, DUP wins $(\mathsf{N}, \mathsf{N} + \mathsf{Z}; n)$.
   -

   And NOW the question. Prove the following rigorously, similar to the end of the slides on DUP SPOILER games.

   (a) For all n, DUP wins the $(\mathsf{N}, \mathsf{N} + \mathsf{Z} + \mathsf{Z}; n)$ game.

   (b) For all n, DUP wins the $(\mathsf{N}, \mathsf{N} + \mathsf{Z} + \mathsf{Z} + \mathsf{Z}; n)$ game. (You can use Part a)

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3. (40 points) Let a_n be defined by

   $a_1 = 10$

   $a_2 = 20$

   $a_3 = 30$

   $(\forall n \geq 4)[a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}]$.

   Using constructive induction find NATURAL NUMBERS A, B such that

   $(\forall n \geq 1)[a_n \leq AB^n]$.

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4. (40 points) In this problem $\frac{n}{2}$ means $\left\lfloor \frac{n}{2} \right\rfloor$. In this problem we will be looking at the recurrence

$a_1 = 1$

$(\forall n \ge 2)[a_n = a_{n-1} + a_{n/2}]$.

(a) (0 points but you will need it for the later parts) Write a program that does the following:

On input d, N determine

For how many $1 \le n \le N$ is $a_n \equiv 0 \pmod{d}$.

For how many $1 \le n \le N$ is $a_n \equiv 1 \pmod{d}$.

For how many $1 \le n \le N$ is $a_n \equiv 2 \pmod{d}$.

$\vdots$

For how many $1 \le n \le N$ is $a_n \equiv d-1 \pmod{d}$.

(Advice: Compute $a_n \pmod{d}$ instead of a_n to avoid large numbers.)

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(b) (20 points) Run your program for $N = 1000$ and $d = 2, 3, \ldots, 20$. Present your data as follows (the numbers below are made up)

$d = 2$

c	$\lvert\{n\colon n \equiv c \pmod 2\}\rvert$
0	410
1	590

$d = 3$

c	$\lvert\{n\colon n \equiv c \pmod 3\}\rvert$
0	333
1	333
2	334

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$d = 20$

c	$\lvert\{n\colon n \equiv c \pmod{20}\}\rvert$
0	100
1	0
2	100
3	0
4	25
5	25
6	25
7	25
8	100
9	0
10	100
11	0
12	25
13	25
14	25
15	25
16	100
17	100
18	200
19	0

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(c) (20 points) Based on your data make a conjecture of the form:

Let c, d be such that $0 \le c \le d-1$ and $d \ge 2$. There exists an infinite number of n such that $a_n \equiv c \pmod{d}$ IFF $XXX(c,d)$.