Homework 9 MORALLY Due Apr 18 at 9:00AM

1. (0 points but please DO IT) What is your name?

2. (30 points) Bill has the following:

   - A fair 6-sided die. So the $\Pr(1) = \cdots = \Pr(6) = \frac{1}{6}$.
   - A bias die with
     $\Pr(1) = \Pr(2) = \Pr(3) = \frac{1}{4}$
     $\Pr(4) = \Pr(5) = \Pr(6) = \frac{1}{12}$

   Emily picks one of these die at random (each with prob $\frac{1}{2}$).

   (a) (15 points) If she rolls it n times and gets n 1's, what is the prob she picked the biased die?

   (b) (15 points) If she rolls it n times and gets n 6's what is the prob she picked the biased die?

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3. (25 points) Emily tosses m balls into n boxes at random. Assume $m \ll n$.

   (a) (15 points) What is the probability that at least FOUR balls are in the same box. (You may use the approximations we used for the problem of THREE balls.)

   (b) (10 points) Let n be fixed and large. Fill in the following statement:

   If $m = XXX$ then the prob of having 4 people in a room is OVER $\frac{1}{2}$ and m is close to the least such value of m. (HINT: Use Part 1 of this problem.)

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4. (25 points) Do a COMBINATORIAL PROOF (NOT algebraic, NOT by induction) for the following statement:

   For all $n \geq 0$, $\sum_{s=0}^{n} \binom{n}{s} 2^s = 3^n$.

   (HINT: The Right Hand Side is the answer to the question:

   How many ways can you 3-color $\{1, \ldots, n\}$.

   Argue that the Left Hand Side solves this same problem.)

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5. (20 points) We are playing with a normal Earth-poker: 13 ranks, 4 suites, hands of size 5.

   What is the probability that a hand has a flush OR a straight but NOT a straight-flush. Give it both in terms of notation like $\binom{52}{5}$ and an actual number like 0.0414 (to 4 places).