Midterm One, March 9 8:00PM-10:00PM
WARNING: THIS MID IS THREE PAGES LONG!!!!!!!!!!!!!!!!!!

1. (15 points) Let p and q be primes. Let $n = p^2q^3$. Show that, $n^{2/5} \notin \mathsf{Q}$. USE Unique Factorization.

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2. (20 points)

   (a) (7 points-1 point each) Fill in the following:

   0) $0^4 \equiv \qquad$ (mod 8).
   1) $1^4 \equiv \qquad$ (mod 8).
   2) $2^4 \equiv \qquad$ (mod 8).
   3) $3^4 \equiv \qquad$ (mod 8).
   4) $4^4 \equiv \qquad$ (mod 8).
   5) $5^4 \equiv \qquad$ (mod 8).
   6) $6^4 \equiv \qquad$ (mod 8).
   7) $7^4 \equiv \qquad$ (mod 8).

   (b) (13 points) Show that there exists an infinite number of n such that n cannot be written as the sum of 6 fourth powers. (HINT: Use Part a.)

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3. (15 points) Find a number M such that the following is true, and prove it.

$$(\forall n \geq M)(\exists x, y \in \mathsf{N})[n = 37x + 38y].$$