Midterm Two, April 20 8:00PM-10:15PM on Zoom
WARNING: THIS MID IS THREE PAGES LONG!!!!!!!!!!!!!!!!

1. (15 points)

   (a) (5 points) What is the coefficient of $x^2y^3z^4$ in

   $$(x+y+z)^9$$

   (b) (10 points) What is the coefficient of $x^ay^bz^c$ in

   $$(x+y+z)^{a+b+c}$$

GO TO NEXT PAGE

2. (20 points–5 points each) The Narns play card games with a cards that have ranks in the set $\{1, 2, \ldots, r\}$ and suites in the set $\{1, \ldots, s\}$. In Narn Poker, each player gets h cards.

   We assume that both r and s are squares, so $\sqrt{r}$ and $\sqrt{s}$ are natural numbers.

   (a) A *Square Hand* is a hand where all of the cards have square rank.

   What is the probability of getting a Square Hand?

   (Its okay if they are of the same suite, or not.)

   (b) A *Square Flush* is a hand where all of the cards have a square rank and all of the suites are the same.

   What is the probability of getting a Square Flush?

   (c) An *Apple* is when you get two of the same rank. There are no other restrictions, so for example, if you had 3 of the same rank, that would still be an Apple.

   What is the probability of getting an Apple.

GO TO NEXT PAGE

3. (15 points) In this problem we guide you through the birthday paradox with m balls in n boxes where we want the probability that at least k balls go in the same box is $\geq \frac{1}{2}$. (HINT: Follow the proof for THREE balls in a box and feel free to use the approximations I use there.)

   Assume that m is much less than n. Assume that k is much less than both n, m.

   We put m balls into n boxes at random.

   (a) Let $i_1, \ldots, i_k$ be k balls. What is the probability they are all in the same box?

   (b) What is the (approx) probability that NO set of k is in the same box? (Use three approximations here: (a) that the events are independent, and (b) use $(1-x)$ is approximately e^{-x}, and (c) $\binom{m}{k} \sim \frac{m^k}{k!}$.

   (c) Think of n, k as being fixed but m as being varrying. Approximatly how large does m have to be so that the prob that k are in the same box is $\geq \frac{1}{2}$?