Ferrers Diagrams

250H

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Notation

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- Note: 2^2 does not mean 2 squared.
  - Instead it means how many times that number appears in a partition.
  - Don’t use this because its confusing, but if you see it out in the wild this is what it means.

Back to Making Change

We only want to look at Pennies and Nickels.

Let $p(0) = p(1) = p(2) = p(3) = p(4) = 1$. Let $n \geq 5$.

$p(n)$: use a nickel or don't

- If we use a nickel then $p(n-5)$
  - makes sense since $n \geq 5$
- If we do not use nickels then you have n cents, only pennies, so 1 way
  - $p(n) = p(n-5)+1$

Everyone's Favorite Forced Social Time

- In breakout rooms, try and find a pattern for large n’s

Pattern

if $0 \leq i \leq 4$:

- $p(5n+i)$ we have to use i pennies, so this is $p(5n)$
- $p(5n) = p(5(n-1)) + 1$
- $p(5n+i) = n+1$

Another Answer

Coefficients of x^n in:

$$(1+x+x^2+\dots)(1+x^5+x^{10}+\dots)$$

$$\left(\frac{1}{1-x}\right)\left(\frac{1}{(1-x^5)}\right) = \frac{1}{(1-x)(1-x^5)} = \frac{1}{x^6-x^5-x+1}$$

$$\frac{1}{(x^6-x^5-x+1)} = 1+x+x^2+x^3+x^4+2x^5+2x^6+2x^7+2x^8+2x^9+3x^{10}+3x^{11}+\dots$$

This is an odd approach to finding taylor series: If we want to find the taylor series of a function, first find a change problem that it answers, solve that change problem, and you have the taylor series

Partitions

Partitions can be viewed graphically with Ferrer Diagrams

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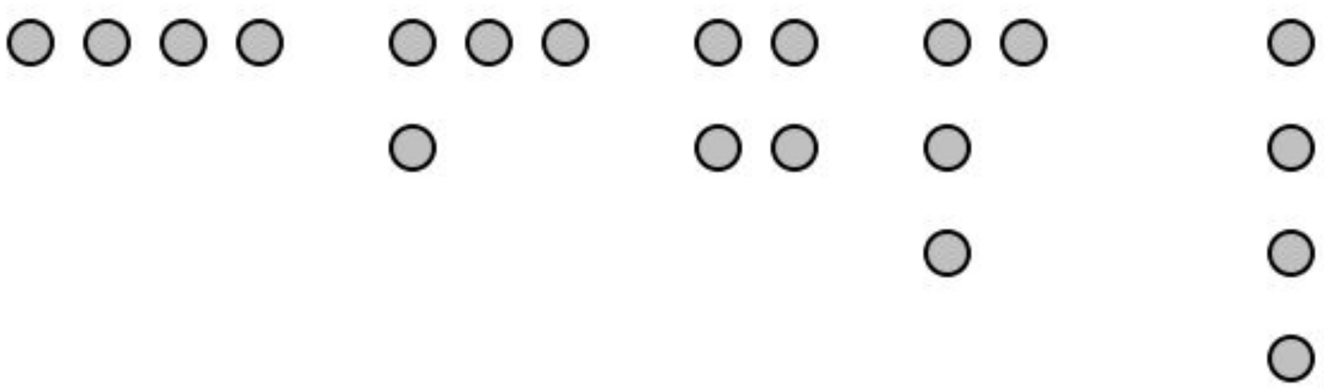
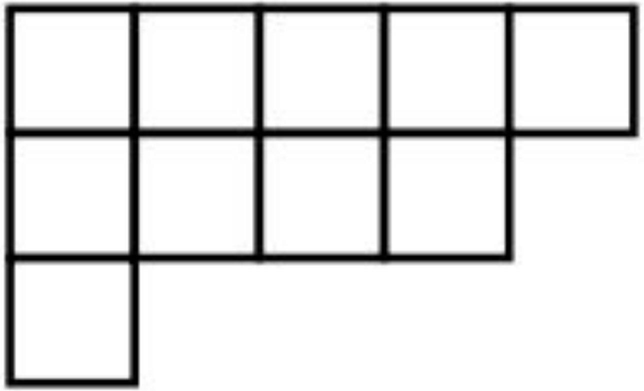
Let’s look at 4’s Ferrer Diagrams.



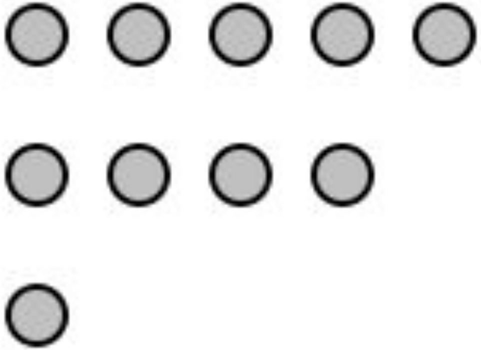
How to make a Ferrer Diagram

- Finite collection of boxes or dots
- Arranged in Left-Justified Rows
- Row Lengths in non-increasing order

Notation



Young Diagram



Ferrers Diagram

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Young Diagrams are useful in

- Symmetric functions and group representation theory
- Polyominoes

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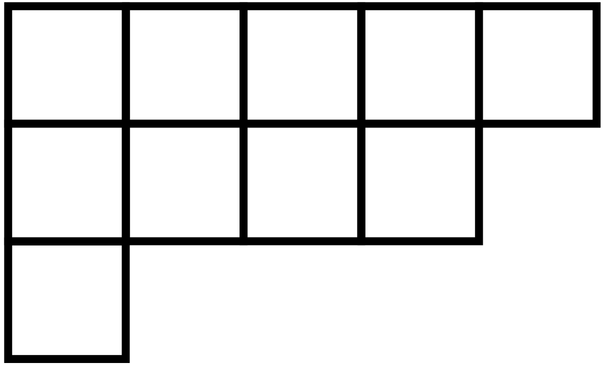
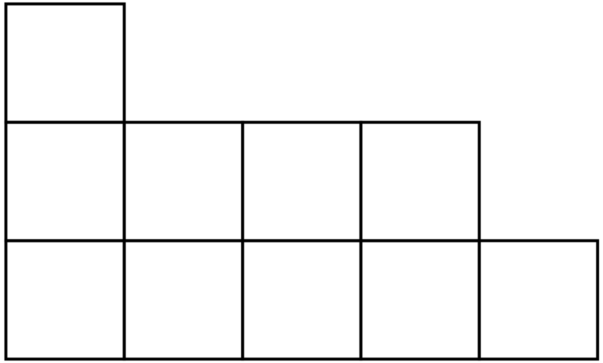


English Notation



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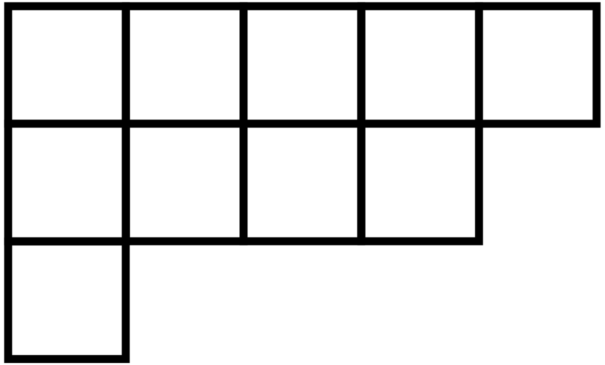

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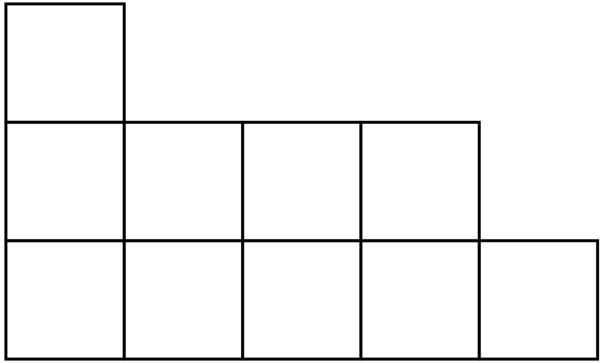

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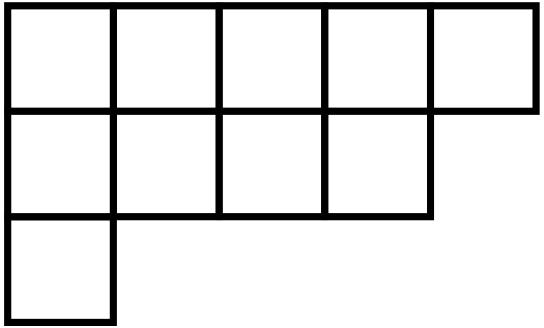

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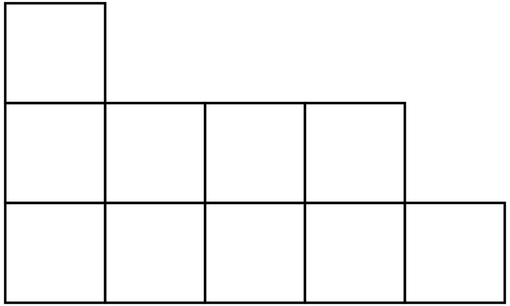

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In his book “symmetric functions”, Macdonald tells readers preferring the French convention to "read this book upside down in a mirror" (Macdonald 1979, p. 2)

Theorem

- The number of ways to partition n into $< m$ parts is the number of ways to partition n into parts the largest of which is $< m$

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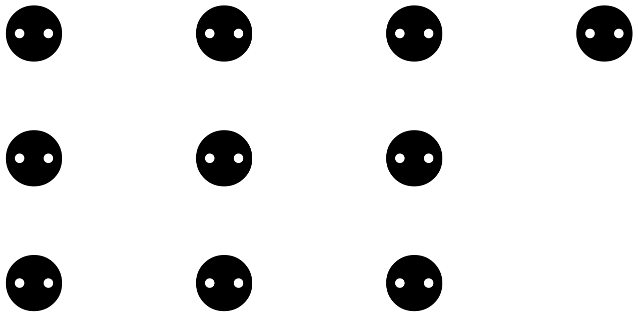
- The number of ways of partitioning n into m parts is equal to the number of ways of partitioning n into parts, the largest of which is m.

Let's look at 10 with n = 3



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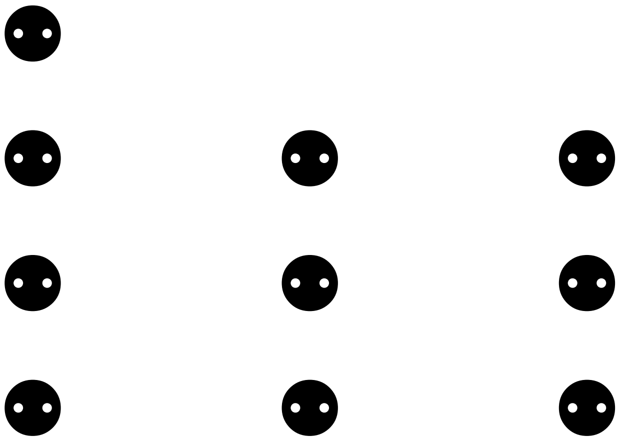

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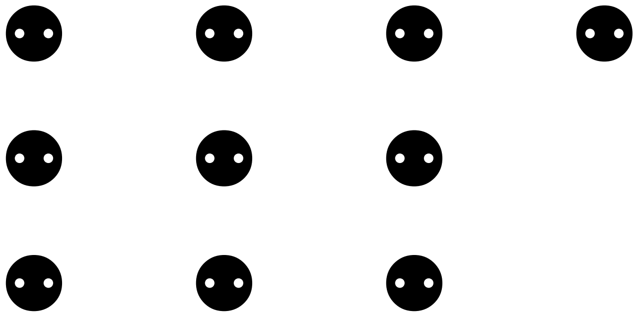

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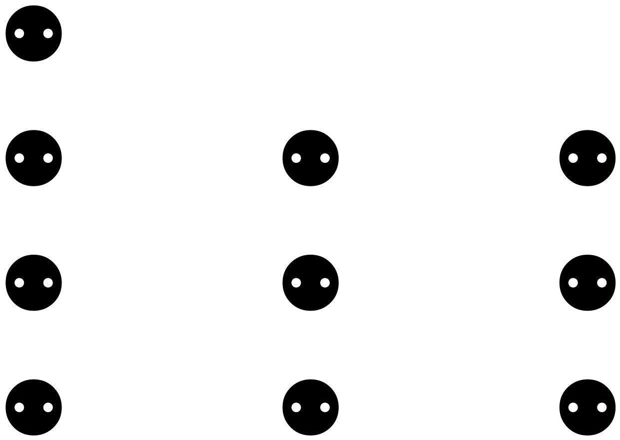

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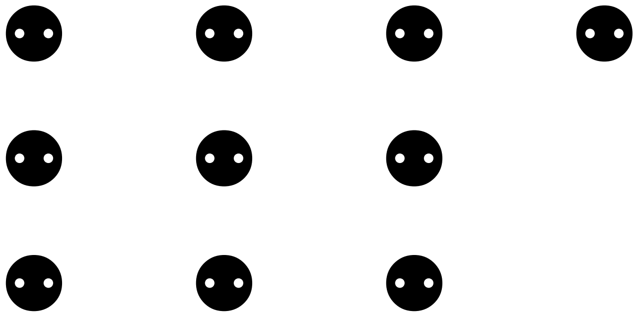

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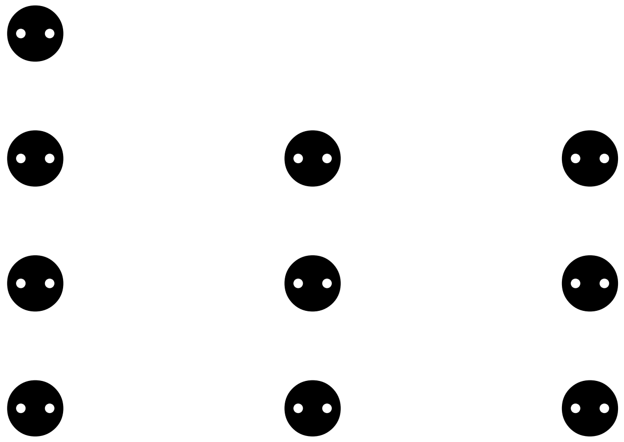

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Theorem: The number of partitions of n into parts that are both odd and distinct is equal to the number of self-conjugate partitions of n. (This can be proved using Ferrer diagrams)