May 10, 2022

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1) Teachers read their teaching eval comments for self-improvement.

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2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.

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5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.

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6) The biggest problem we have for all of the above is when not that many students fill them out. Hence

1) Teachers read their teaching eval comments for self-improvement.

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7) Please Fill Out the Teaching Evals in All of your Courses

# How to Use the Half Method (1) Given Upper Bound (2) NOT Given Upper Bound

May 10, 2022

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# How Your Program Shows $f(45, 26) \leq \frac{32}{78}$

May 10, 2022

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$$f(m,s) \leq \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s} \times \frac{1}{\lceil 2m/s \rceil}, 1 - \frac{m}{2} \times \frac{1}{\lfloor 2m/s \rfloor}\right\}\right\}.$$

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$$f(m,s) \le \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s} \times \frac{1}{\lceil 2m/s \rceil}, 1 - \frac{m}{2} \times \frac{1}{\lfloor 2m/s \rfloor}\right\}\right\}.$$
$$\frac{2m}{s} = \frac{90}{26} = \frac{45}{13} \sim 3.46 \qquad \left\lfloor\frac{2m}{s}\right\rfloor = 3 \qquad \left\lceil\frac{2m}{s}\right\rceil = 4$$

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$$f(m,s) \le \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s} \times \frac{1}{\lceil 2m/s \rceil}, 1 - \frac{m}{2} \times \frac{1}{\lfloor 2m/s \rfloor}\right\}\right\}.$$
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$$f(45,26) \le \max\left\{\frac{1}{3}, \min\left\{\frac{45}{26} \times \frac{1}{4}, 1 - \frac{45}{26} \times \frac{1}{3}\right\}\right\} = \min\left\{\frac{45}{104}, 1 - \frac{15}{26}\right\}$$

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$$f(m,s) \le \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s} \times \frac{1}{\lceil 2m/s \rceil}, 1 - \frac{m}{2} \times \frac{1}{\lfloor 2m/s \rfloor}\right\}\right\}.$$

$$\frac{2m}{s} = \frac{90}{26} = \frac{45}{13} \sim 3.46 \qquad \left\lfloor\frac{2m}{s}\right\rfloor = 3 \qquad \left\lceil\frac{2m}{s}\right\rceil = 4$$

$$f(45,26) \le \max\left\{\frac{1}{3}, \min\left\{\frac{45}{26} \times \frac{1}{4}, 1 - \frac{45}{26} \times \frac{1}{3}\right\}\right\} = \min\left\{\frac{45}{104}, 1 - \frac{15}{26}\right\}$$

$$= \min\left\{\frac{45}{104}, \frac{11}{26}\right\} = \frac{11}{26} \sim 0.423$$

We want  $f(m, s) \leq \frac{32}{78} \sim 0.410$ . So FC NOT powerful enough.

Your program will try  $V = 2, 3, 4, \ldots$  and find out that

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Your program will try V = 2, 3, 4, ... and find out that If someone gets  $\leq 2$  shares then  $\exists$  a share  $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$ .

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Your program will try  $V = 2, 3, 4, \ldots$  and find out that If someone gets  $\leq 2$  shares then  $\exists$  a share  $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$ . Buddy is  $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$ .

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Your program will try V = 2, 3, 4, ... and find out that If someone gets  $\leq 2$  shares then  $\exists$  a share  $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$ . Buddy is  $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$ .

If someone gets  $\geq 5$  shares then  $\exists$  a share  $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$ .

Your program will try V = 2, 3, 4, ... and find out that If someone gets  $\leq 2$  shares then  $\exists$  a share  $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$ . Buddy is  $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$ . If someone gets  $\geq 5$  shares then  $\exists$  a share  $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$ .

V = 4.

Your program will try  $V = 2, 3, 4, \ldots$  and find out that If someone gets  $\leq 2$  shares then  $\exists$  a share  $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$ . Buddy is  $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$ . If someone gets  $\geq 5$  shares then  $\exists$  a share  $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$ .

V = 4.

Some students gets 3 shares. Some students gets 4 shares.

 $3s_3 + 4s_4 = 90$  $s_3 + s_4 = 26$ 

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 $3s_3 + 4s_4 = 90$   $s_3 + s_4 = 26$   $s_3 = 14$  $s_4 = 12$ .

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 $3s_3 + 4s_4 = 90$  $s_3 + s_4 = 26$  $s_3 = 14$ 

 $s_4 = 12.$ 

14 students get 3 shares



 $3s_3 + 4s_4 = 90$  $s_3 + s_4 = 26$  $s_3 = 14$ 

 $s_4 = 12.$ 

14 students get 3 shares12 students get 4 shares

 $3s_3 + 4s_4 = 90$  $s_3 + s_4 = 26$ 

 $s_3 = 14$  $s_4 = 12.$ 

14 students get 3 shares 12 students get 4 shares Note: there are  $3 \times 14 = 42$  3-shares there are  $4 \times 12 = 48$  4-shares.

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 $3s_3 + 4s_4 = 90$  $s_3 + s_4 = 26$ 

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14 students get 3 shares 12 students get 4 shares Note: there are  $3 \times 14 = 42$  3-shares there are  $4 \times 12 = 48$  4-shares.

**Note** One way for HALF to not work is if these equations have a solution that is not in  $\mathbb{N}$ .

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Want if  $\exists$  4-share  $\geq \beta$  then some piece  $\leq \frac{32}{78}$ .

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**Want** if  $\exists$  4-share  $\geq \beta$  then some piece  $\leq \frac{32}{78}$ .

Alice has  $p_1 \leq p_2 \leq p_3 \leq p_4$ .

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Want if  $\exists$  4-share  $\geq \beta$  then some piece  $\leq \frac{32}{78}$ .

Alice has  $p_1 \le p_2 \le p_3 \le p_4$ . Assume  $p_4 \ge \beta$  and later pick  $\beta$  to get a contradiction.  $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$  $p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \le \frac{45}{26} - \beta$ 

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**Want** if  $\exists$  4-share  $\geq \beta$  then some piece  $\leq \frac{32}{78}$ .

Alice has  $p_1 \le p_2 \le p_3 \le p_4$ . Assume  $p_4 \ge \beta$  and later pick  $\beta$  to get a contradiction.  $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$   $p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \le \frac{45}{26} - \beta$   $p_1 \le \frac{1}{3}(\frac{45}{26} - \beta)$ . Want  $\beta$  so that  $p_1 \le \frac{32}{78}$ :  $\frac{1}{3}(\frac{45}{26} - \beta) \le \frac{32}{78}$ 

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**Want** if  $\exists$  4-share  $\geq \beta$  then some piece  $\leq \frac{32}{78}$ .

Alice has  $p_1 \leq p_2 \leq p_3 \leq p_4$ . Assume  $p_4 \geq \beta$  and later pick  $\beta$  to get a contradiction.  $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$   $p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \beta$   $p_1 \leq \frac{1}{3}(\frac{45}{26} - \beta)$ . Want  $\beta$  so that  $p_1 \leq \frac{32}{78}$ :  $\frac{1}{3}(\frac{45}{26} - \beta) \leq \frac{32}{78}$  $\beta \geq \frac{39}{78} = \frac{1}{2}$ . Take  $\beta = \frac{39}{78}$ .

# The $\gamma$ Step (What you Really Want to See)

## The $\gamma$ Step (What you Really Want to See)

Want if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ .
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Want if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ . Bob has  $p_1 \leq p_2 \leq p_3$ .

Want if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ .

Bob has  $p_1 \leq p_2 \leq p_3$ . Assume  $p_1 \leq \gamma$  and later pick  $\gamma$  to get a contradiction.  $p_1 + p_2 + p_3 = \frac{45}{26}$  $p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma$ 

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**Want** if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ .

Bob has  $p_1 \leq p_2 \leq p_3$ . Assume  $p_1 \leq \gamma$  and later pick  $\gamma$  to get a contradiction.  $p_1 + p_2 + p_3 = \frac{45}{26}$  $p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma$  $p_3 \geq \frac{1}{2}(\frac{45}{26} - \gamma)$ . Key Look at buddy of  $p_3$ .

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**Want** if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ .

Bob has  $p_1 \le p_2 \le p_3$ . Assume  $p_1 \le \gamma$  and later pick  $\gamma$  to get a contradiction.  $p_1 + p_2 + p_3 = \frac{45}{26}$   $p_2 + p_3 = \frac{45}{26} - p_1 \ge \frac{45}{26} - \gamma$   $p_3 \ge \frac{1}{2}(\frac{45}{26} - \gamma)$ . Key Look at buddy of  $p_3$ .  $1 - p_3 \le 1 - \frac{1}{2}(\frac{45}{26} - \gamma)$ . Want  $\gamma$  so that  $1 - p_3 \le \frac{32}{78}$ :

**Want** if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ .

Bob has  $p_1 \le p_2 \le p_3$ . Assume  $p_1 \le \gamma$  and later pick  $\gamma$  to get a contradiction.  $p_1 + p_2 + p_3 = \frac{45}{26}$   $p_2 + p_3 = \frac{45}{26} - p_1 \ge \frac{45}{26} - \gamma$   $p_3 \ge \frac{1}{2}(\frac{45}{26} - \gamma)$ . Key Look at buddy of  $p_3$ .  $1 - p_3 \le 1 - \frac{1}{2}(\frac{45}{26} - \gamma)$ . Want  $\gamma$  so that  $1 - p_3 \le \frac{32}{78}$ :  $1 - \frac{1}{2}(\frac{45}{26} - \gamma) \le \frac{32}{78}$ .

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**Want** if  $\exists$  3-share  $\leq \gamma$  then some piece  $\leq \frac{32}{78}$ .

Bob has  $p_1 \leq p_2 \leq p_3$ . Assume  $p_1 \leq \gamma$  and later pick  $\gamma$  to get a contradiction.  $p_1 + p_2 + p_3 = \frac{45}{26}$   $p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma$   $p_3 \geq \frac{1}{2}(\frac{45}{26} - \gamma)$ . Key Look at buddy of  $p_3$ .  $1 - p_3 \leq 1 - \frac{1}{2}(\frac{45}{26} - \gamma)$ . Want  $\gamma$  so that  $1 - p_3 \leq \frac{32}{78}$ :  $1 - \frac{1}{2}(\frac{45}{26} - \gamma) \leq \frac{32}{78}$ .  $\gamma \leq \frac{43}{78}$ . Take  $\gamma = \frac{43}{78}$ .

#### **VHALF Step**

There are 42 3-shares. There are 48 4-shares

$$\beta = \frac{1}{2} \leq \frac{1}{2} \leq \gamma = \frac{43}{78}$$
 42  $\neq$  48 so SUCCESS!

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#### VHALF Step

There are 42 3-shares. There are 48 4-shares

$$\beta = \frac{1}{2} \le \frac{1}{2} \le \gamma = \frac{43}{78}$$

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 $42 \neq 48$  so SUCCESS!

**Note** One way for the HALF method to fail is if  $\gamma < \beta$ 

# How Your Program Finds the Answer

May 10, 2022

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$$V = \left\lceil \frac{2m}{s} \right\rceil = \left\lceil \frac{90}{26} \right\rceil = 4.$$

$$V = \left\lceil \frac{2m}{s} \right\rceil = \left\lceil \frac{90}{26} \right\rceil = 4.$$
  
$$s_3 = 14$$

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$$V = \left\lceil \frac{2m}{s} \right\rceil = \left\lceil \frac{90}{26} \right\rceil = 4.$$
  

$$s_3 = 14$$
  

$$s_4 = 12.$$

$$\begin{array}{cccc} ( & 48 \text{ 4-shs} & )[ & 0 & ]( & 42 \text{ 3-shs} & ) \\ \alpha & \beta & \gamma & 1-\alpha \end{array}$$

$$\begin{array}{cccc} ( & 48 \text{ 4-shs} & )[ & 0 & ]( & 42 \text{ 3-shs} & ) \\ \alpha & \beta & \gamma & 1-\alpha \end{array}$$

We want to set  $\alpha$  so that  $\beta = \frac{1}{2}$ . Alice has  $p_1 \leq p_2 \leq p_3 \leq p_4$ .

$$\begin{array}{cccc} ( & 48 \text{ 4-shs} & )[ & 0 & ]( & 42 \text{ 3-shs} & ) \\ \alpha & & \beta & \gamma & & 1-\alpha \end{array}$$

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We want to set  $\alpha$  so that  $\beta = \frac{1}{2}$ .

Alice has 
$$p_1 \le p_2 \le p_3 \le p_4$$
.  
Assume  $p_4 \ge \beta = \frac{1}{2}$  and later pick  $\alpha$  to get a contradiction.  
 $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$   
 $p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \le \frac{45}{26} - \frac{1}{2}$ 

$$\begin{array}{cccc} ( \begin{array}{cccc} 48 \ \text{4-shs} \end{array}) [ \begin{array}{cccc} 0 \end{array} ] ( \begin{array}{cccc} 42 \ \text{3-shs} \end{array} ) \\ \alpha & \beta & \gamma & 1-\alpha \end{array}$$

We want to set  $\alpha$  so that  $\beta = \frac{1}{2}$ .

Alice has 
$$p_1 \le p_2 \le p_3 \le p_4$$
.  
Assume  $p_4 \ge \beta = \frac{1}{2}$  and later pick  $\alpha$  to get a contradiction  
 $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$   
 $p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \le \frac{45}{26} - \frac{1}{2}$   
 $p_1 \le \frac{1}{3}(\frac{45}{26} - \frac{1}{2})$ . Want  $\alpha$  so that  $p_1 \le \alpha$ :

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$$\begin{array}{cccc} ( \begin{array}{cccc} 48 \ \text{4-shs} \end{array}) [ \begin{array}{cccc} 0 \end{array} ] ( \begin{array}{cccc} 42 \ \text{3-shs} \end{array} ) \\ \alpha & \beta & \gamma & 1-\alpha \end{array}$$

We want to set  $\alpha$  so that  $\beta = \frac{1}{2}$ .

Alice has  $p_1 \le p_2 \le p_3 \le p_4$ . Assume  $p_4 \ge \beta = \frac{1}{2}$  and later pick  $\alpha$  to get a contradiction.  $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$   $p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \le \frac{45}{26} - \frac{1}{2}$   $p_1 \le \frac{1}{3}(\frac{45}{26} - \frac{1}{2})$ . Want  $\alpha$  so that  $p_1 \le \alpha$ :  $\frac{1}{3}(\frac{45}{26} - \frac{1}{2}) = \frac{32}{78}$ . We note that there are 48 > 45 4-shares that are all  $\le \frac{1}{2}$ . So we have  $\frac{32}{78}$  is an upper bound.

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Bob has  $p_1 \leq p_2 \leq p_3$ . Assume  $p_1 \leq \frac{1}{2}$  and later pick  $\alpha$  to get a contradiction.  $p_1 + p_2 + p_3 = \frac{45}{26}$  $p_2 + p_3 = \frac{45}{26} - \frac{1}{2}$ 

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 $p_3 \geq \frac{1}{2}(\frac{45}{26} - \frac{1}{2})$ . Key Look at buddy of  $p_3$ .

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 $1 - \frac{1}{2}(\frac{45}{26} - \frac{1}{2}) = \frac{5}{13}$ .

#### **Final Step**

#### We get that both $\frac{32}{78}$ and $\frac{5}{13}$ are potential upper bounds.

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Run *VHALF* on both of them to find out that  $\frac{5}{13}$  is not an upper bound.

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We get that both  $\frac{32}{78}$  and  $\frac{5}{13}$  are potential upper bounds.

Run *VHALF* on both of them to find out that  $\frac{5}{13}$  is not an upper bound.

So answer is  $\frac{32}{78}$ .