On the number of solutions to equations

250H

How many solutions are there to $x_1 + \ldots + x_k = n$, where $x_1, \ldots, x_k$ in $\{0,1,2,\ldots\}$?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

Balls and Lines \ Stars and Bars

How many ways there are to put n indistinguishable balls into k distinguishable bins?

*|***|**

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$$\frac{(n+k-1)!}{n!(k-1)!} = \binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

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$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

$$\frac{(n+k-1)!}{n!(k-1)!} = \frac{(40+5-1)!}{40!(5-1)!} = 135751$$

Stars and bars: What if all the x_i are ≥ 1?

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$$\frac{((n-k)+k-1)!}{(n-k)!(k-1)!} = \frac{n-1!}{(n-k)!(k-1)!}$$