The Emptier-Filler Game

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E: The Emptier
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There will be a bin with numbers in it.

- If the bin is ever empty then E wins.
- If game goes forever and bin is always nonempty then F wins.

The Emptier-Filler Game on $\mathbb{N}$

1) F puts a **finite** multiset of $\mathbb{N}$ into the bin.
(e.g., bin has $\{1, 1, 1, 2, 3, 4, 9, 9, 18, 18\}$.

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(e.g., replace 18 with 99,999,999 17's and 5000 16's.)

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(e.g., replace 18 with 99,999,999 17's and 5000 16's.)

Which player has the winning strategy? What is that strategy.
Breakout Rooms!

Answer!

E wins!

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Strategy for E Keep removing the largest number in the box.

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Answer on next slide.

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- If $n \geq 1$ then the ordered pair is now $(r, n-1)$.
- If $n = 0$ then their are no balls of rank r. Let the highest rank be $r' < r$. Assume there are n' balls of rank r'. Then the ordered pair is now (r', n').

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Consider the following funky ordering on ordered pairs.

$$(0,0) < (0,1) < (0,2) < \cdots < (1,0) < (1,1) < (1,2) < \cdots\cdots .$$

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This is the ordering to use since this quantity always decreases.

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As noted in the last slide if E removes the top ranked ball and F puts in as many balls of lower rank, then the resulting position is associated to $(n', r') < (n, r)$.

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But Can we do induction on this funky ordering?

The Funky Ordering

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$\mathbb{N}$ It only used that if you start at some n and march downward you will **in a finite number of steps** get to 0. In fact, just n steps.

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Yes. However there is no bound on that number of steps. But that does not matter.

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Yes. However there is no bound on that number of steps. But that does not matter.

Def An ordering is **well ordered** if when you start at any element x and march downward you will get to a MIN element in a finite number of steps.

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Upshot You can do induction on any well ordered ordering.

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What if E plays differently? One can show that **no matter what E does, she wins!**

How to prove that? By an induction on a an even funkier ordering. We won't be doing that.

The Emptier-Filler Game on Other Orderings

X is any of $\mathbb{Z}$, $\mathbb{Q}^{\geq 0}$, $\mathbb{N}+\mathbb{N}$, $\mathbb{N}+\mathbb{N}+\cdots$, $\mathbb{N}+\mathbb{Z}$, $\mathbb{N}+\mathbb{N}^*$.
1) F puts a **finite** multiset of X into the bin.

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For each of $X=\mathbb{Z}$, $X=\mathbb{Q}$, $X=\mathbb{N}+N$, $X=\mathbb{N}+\mathbb{N}+\cdots$, $X=\mathbb{N}+\mathbb{Z}$, $X=\mathbb{N}+\mathbb{N}^*$ who wins?
Breakout Rooms!

Answers!

$\boldsymbol{X} = \mathbb{N}$

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Need a General Theorem

Question Let X be a set and $\preceq$ be an ordering on it. Let the $(X, \preceq)$-game be the game as above where we put elements of X in the bin.

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In the following sentence fill in the ???
E can win the $(X, \preceq)$-game if and only if $(X, \preceq)$ has property ???.
Breakout Rooms!

Answer!

Def $(X, \preceq)$ is **well ordered** if there are NO infinite decreasing sequences.

Answer!

Def $(X, \preceq)$ is **well ordered** if there are NO infinite decreasing sequences.

E can win the $(X, \preceq)$-game if and only if $(X, \preceq)$ is well ordered.