

# Intermediary Truth Values

# TRUE is 1, FALSE is 0

We have dealt with **math** where statement really are **true** or **false** (with some rare exceptions).

The **Real World** is messier!

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- ▶ Its going to rain tomorrow. Forecasts give probabilities.



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We will **not** dwell on this. We will ponder a serious math question about intermediary truth values.

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2. Want the definitions to satisfy De Morgan' Law.
3. Want the definitions to make sense intuitively. For example  $x \wedge y \leq x$  and  $x \vee y \geq x$ .

Work on in groups!

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