

SATisfiability

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3. Is this problem interesting to people outside of Logic?

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UNKNOWN TO SCIENCE If there are no restrictions on the formula, then unknown if there is an algorithm better than $\sim 2^n$.

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Notation We denote Polynomial Time by **P**.

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How Long Has It Been Open For? First posed in 1971, though see next slide.

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- ▶ The complexity of 3-SAT is **important** since it relates to the complexity of many other problems.
- ▶ Many of the problems 3-SAT is equivalent to have been worked on for 90 or more years; hence, it is unlikely they are in P. Hence it is unlikely that 3-SAT is in P.

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Its not all Bad News I

Scenario Your boss wants you to solve the TSP problem. You know that finding the **optimal** solution is likely not easy to do. So you know to look for an **approximation**. Perhaps something that is at worst twice optimal.

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More generally, if you now a problem is equivalent to SAT then you know that you should not look for an optimal poly time solutions. There are many other options to try.

Its not all Bad News II

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$$4 + 4 = 8$$

$$8 + 8 = 16$$

$$16 + 16 = 32$$

$$32 + 32 = 64$$

$$64 + 64 = 128$$

$$128 + 128 = 256$$

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He then ran out of room; however, his grandmother (my wife's sister) tells me he can go all the way to 2048.