

# SATisfiability

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2. Is there a class of formulas for which there is a better algorithm?
3. Is this problem interesting to people outside of Logic?

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**YES** If  $\phi$  is in 3-CNF form (we'll define that later) then there exists a randomized  $1.306^n$  algorithm.

**UNKNOWN TO SCIENCE** If there are no restrictions on the formula, then unknown if there is an algorithm better than  $\sim 2^n$ .

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However, the  $n^{100}$  algorithm **is not doing brute force search!**

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**Notation** We denote Polynomial Time by **P**.

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- ▶ Otherwise  $\phi \notin \text{DNFSAT}$ .

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**How Long Has It Been Open For?** First posed in 1971, though see next slide.

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Consider the following problems:

1. **Traveling Salesperson Problem (TSP)** Given  $n$  cities and how much it costs to go from any city to an city, determine cheapest way to visit all cities, Studied since the 1930's.
2. **Scheduling** Given  $n$  rooms and when the free, and given  $m$  people who are requesting them for certain timeslots, can you accommodates all of them? Studied since the 1880's.

The following is known:

$(3\text{-SAT is in P}) \leftrightarrow (\text{TSP is in P}) \leftrightarrow (\text{SCHED is in P})$ .

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- ▶ The complexity of 3-SAT is **important** since it relates to the complexity of many other problems.
- ▶ Many of the problems 3-SAT is equivalent to have been worked on for 90 or more years; hence, it is unlikely they are in P. Hence it is unlikely that 3-SAT is in P.

# Proper Terminology and What Do People In the Know Think?

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More generally, if you now a problem is equivalent to SAT then you know that you should not look for an optimal poly time solutions. There are many other options to try.

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He then ran out of room; however, his grandmother (my wife's sister) tells me he can go all the way to 2048.