START RECORDING

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- True History: Approximations of the above.

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- More generally, if S is any statement then

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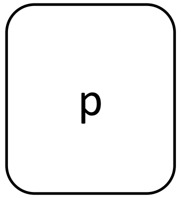
  is true.
- Aristotle and others thought that using Logic they could settle arguments in philosophy and other fields.
- We know better.

Module 1: Propositional Logic

- The most elementary kind of logic in Computer Science
- Also known as Boolean Logic, by virtue of *George Boole* (1815 – 1864)

Propositional Symbols

- The building blocks of propositional logic.
- Think of them as **bits** or **boxes** that hold a value of 1 (True) or 0 (False)
- Denoted using a lowercase English letter (p, q, … , z)



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    - IS proposition. Also its TRUE.
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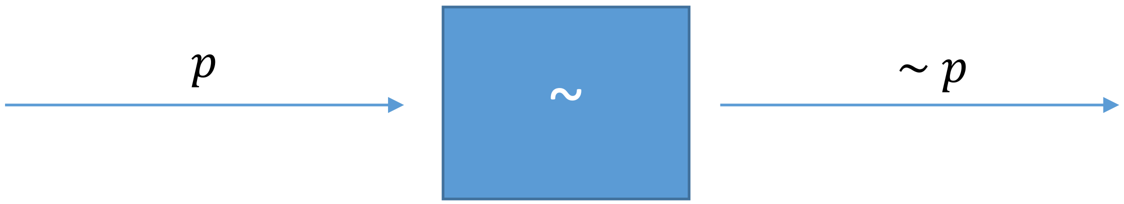
    - IS a proposition whether or not it is true.
  - $2 + 2 = 5$
    - YES its a proposition. Its FALSE.

Operations in Boolean logic

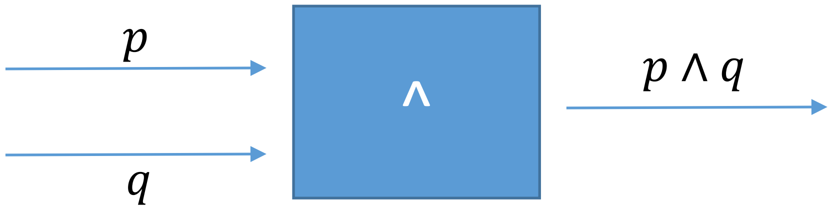
- There are three basic operations in boolean logic
  - Conjunction (AND)
  - Disjunction (OR)
  - Negation (NOT)
- Other operations can be defined *in terms of those three.*

Negation (NOT, ~, ¬)



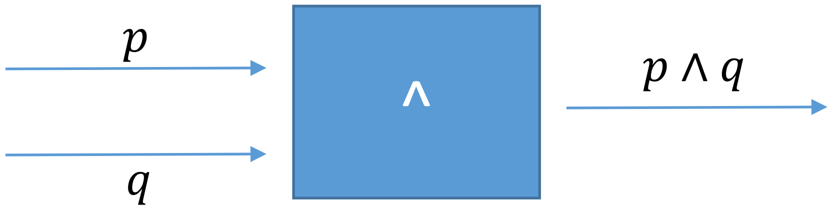
p	$\sim p$
F	***T***
T	***F***

Conjunction (^)



p	q	$p \wedge q$
F	***F***	***F***
F	***T***	***F***
T	***F***	***F***
T	***T***	***T***

Conjunction (^)



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F	***F***	***F***
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T	***F***	***F***
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Rule of thumb: p **<u>and</u>** q must be 1

Fun exercise

- Fill-in the following truth table:

p	q	$p \wedge (\sim q)$
F	***F***	***?***
F	***T***	***?***
T	***F***	***?***
T	***T***	***?***

Fun exercise

- Fill-in the following truth table:

p	q	$p \wedge (\sim q)$
F	*F*	
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T	*F*	
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- Fill-in the following truth table:

p	q	$p \land (\sim q)$
F	***F***	***F***
F	***T***	
T	***F***	
T	***T***	

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Disjunction

p	q	$p \vee q$
F	***F***	***F***
F	***T***	***T***
T	***F***	***T***
T	***T***	***T***

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<u>**one of**</u> p <u>**or**</u> q
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p	q	$p \vee (p \wedge q)$
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F	***T***	***?***
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Implication

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- WARNING: This will NOT be like how we use implication IRL.
  - IRL we use implication to mean that P really helps you to establish Q.
  - That will not be the case here.

Examples and Intuition of Implication

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  - If the moon is made of green cheese then $2 + 2 = 5$

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  - If the moon is made of green cheese then $2 + 2 = 4$

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- UPSHOT: In truth table for $p \rightarrow q$ whenever p is FALSE $p \rightarrow q$ will be TRUE

More Examples and Intuitions of Implication

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- What case is left?
  - If $2 + 2 = 4$ then Emily is 6 feet tall.
    - FALSE- a TRUE statement cannot imply a FALSE statement.

Truth Table for Implication ($\Longrightarrow$)

- "If –then"

p	q	$p \Rightarrow q$
F	***F***	***T***
F	***T***	***T***
T	***F***	***F***
T	***T***	***T***

Bi-conditional ($\Leftrightarrow$)

- "If and only if"

p	q	$p \Leftrightarrow q$
F	***F***	***T***
F	***T***	***F***
T	***F***	***F***
T	***T***	***T***

Practice

- Fill in the following truth tables:

p	$p \implies (\sim p)$
F	***?***
T	***?***

p	q	r	$(p \wedge q) \Rightarrow r$
F	***F***	***F***	***?***
F	***F***	***T***	***?***
F	***T***	***F***	***?***
F	***T***	***T***	***?***
T	***F***	***F***	***?***
T	***F***	***T***	***?***
T	***T***	***F***	***?***
T	***T***	***T***	***?***

Contradictions / Tautologies

- Examine the statements:
  - $p \wedge (\sim p)$
  - $p \vee (\sim p)$
- What can you say about those statements?

STOP RECORDING