Recall the first theorem one usually hears in Ramsey Theory and can tell your non-math friends about.

*If there are 6 people at a party, either 3 know each other or 3 do not know each other.*
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*If there are 6 people at a party, either 3 know each other or 3 do not know each other.*

We define graphs and complete graphs and state this theorem in those terms.
Graphs and Complete Graphs

**Def** A **Graph** $G = (V, E)$ is a set $V$ and a set of unordered pairs from $V$, called edges. These can easily be drawn.

Example

$V = \{1, 2, 3, 4, 5, 6\}$.

$E = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)\}$. 

**Def** The degree (deg) of a vertex is how many edges use it.

In the above graph $\text{deg}(1) = 5$ and $\text{deg}(2) = \text{deg}(3) = \text{deg}(4) = \text{deg}(5) = \text{deg}(6) = 1$. 
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**Example**

![Graph diagram](image)

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**Example**

```
1

2 3 4
```

This graph is $K_4$. 

Note Every vertex of $K_n$ has degree $n - 1$. 

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Below is standard notation which you may or may not have seen.
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- ∃ means **there exists**
More Notation

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**Notation**

- \( \exists \) means *there exists*
- \( \forall \) means *for all*
One More Definition

**Def** Let $G = (V, E)$ be a graph. Let $U \subseteq V$.

1. $U$ is a **Clique** if all of the verts in $U$ have an edge between them.
2. If $|U| = k$ then we may call $U$ a **$k$-clique**.
3. If the edges of $G$ are 2-colored with RED and BLUE, and all of the edges between verts of $U$ are RED then we call $U$ a **Red Clique**. Similar for Blue.

4. If I formed a rock band it would be called **Bill Gasarch and the Red Cliques**!
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For every 2-coloring of the edges of $K_6$ there is a monochromatic $K_3$ (triangle).
The First Theorem, Restated

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We prove this in the next few slides.
Focus on Vertex 1

Given a 2-coloring of the edges of $K_6$ we look at vertex 1.
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Given a 2-coloring of the edges of $K_6$ we look at vertex 1.

There are 5 edges coming out of vertex 1. They are 2 colored. There exist 3 edges from vertex 1 that are the same color. We can assume (1, 2), (1, 3), (1, 4) are all RED.
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(1,2), (1,3), (1,4) are RED
We Look Just at Vertices 1, 2, 3, 4

If (2, 3) is RED then get RED Triangle. So assume (2, 3) is BLUE.
We Look Just at Vertices 1,2,3,4

If (2, 3) is **RED** then get **RED** Triangle. So assume (2,3) is **BLUE**.
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If (2,4) is **RED** then get **RED** triangle. So assume (2,4) is **BLUE**.
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Note that there is a **BLUE** triangle with verts 2, 3, 4. Done!
What if we color edges of $K_5$?

This graph is not arbitrary. $SQ_5 = \{x^2 \pmod{5} : 0 \leq x \leq 4\} = \{0, 1, 4\}$.

$\Rightarrow$ If $i - j \in SQ_5$ then RED.

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- If $i - j \in SQ_5$ then **RED**.
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Asymmetric Ramsey Numbers

Definition $R(a, b)$ is least $n$ such that for all 2-colorings of $K_n$ there is either a red $K_a$ or a blue $K_b$.

1. $R(a, b) = R(b, a)$.
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Proof left to the reader, but its easy.
Theorem \( R(a, b) \leq R(a - 1, b) + R(a, b - 1) \)
Theorem $R(a, b) \leq R(a - 1, b) + R(a, b - 1)$

Let $n = R(a - 1, b) + R(a, b - 1)$. 

Assume you have a coloring of the edges of $K_n$. The proof has three cases on the next three slides. They will be:

1. There is a vertex with large Red Deg.
2. There is a vertex with large Blue Deg.
3. All verts have small Red degree and small Blue degree.
\[ R(a, b) \leq R(a - 1, b) + R(a, b - 1) \]

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![Diagram](image)

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![Diagram with vertices and edges]

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Case 3 Negate Case 1 and Case 2:
All Verts: Small Red Deg and Small Blue Deg

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\[(\forall v)[\deg(v) \leq R(a - 1, b) + R(a, b - 1) - 2 = n - 2]\]
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Not possible since every vertex of \(K_n\) has degree \(n - 1\).
Let's compute bounds on $R(a, b)$

- $R(3, 3) \leq R(2, 3) + R(3, 2) \leq 3 + 3 = 6$
- $R(3, 4) \leq R(2, 4) + R(3, 3) \leq 4 + 6 = 10$
- $R(3, 5) \leq R(2, 5) + R(3, 4) \leq 5 + 10 = 15$
- $R(3, 6) \leq R(2, 6) + R(3, 5) \leq 6 + 15 = 21$
- $R(3, 7) \leq R(2, 7) + R(3, 6) \leq 7 + 21 = 28$
- $R(4, 4) \leq R(3, 4) + R(4, 3) \leq 10 + 10 = 20$
- $R(4, 5) \leq R(3, 5) + R(4, 4) \leq 15 + 20 = 35$
- $R(5, 5) \leq R(4, 5) + R(5, 4) \leq 35 + 35 = 70$. 
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Can we make some improvements to this? **YES!**

We need a theorem.
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Can we make some improvements to this? YES! We need a theorem. We first do an example.
A Graph on 9 Vertices with all verts Deg 3?

**Thm** There is NO graph on 9 verts, with every vertex of deg 3.
A Graph on 9 Vertices with all verts Deg 3?

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We generalize this on the next slide.
Handshake Lemma

**Lemma** Let $G = (V, E)$ be a graph.

Then $|V_{\text{odd}}| \equiv 0 \pmod{2}$.

$$\sum_{v \in V_{\text{even}}} \deg(v) + \sum_{v \in V_{\text{odd}}} \deg(v) = \sum_{v \in V} \deg(v) = 2|E| \equiv 0 \pmod{2}.$$
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**Handshake Lemma** If all pairs of people in a room shake hands, even number of shakes. (Pre-COVID when people shook hands.)
Corollary of Handshake Lemma

Impossible to have a graph on an odd number of verts where every vertex is of odd degree.
Corollary of Handshake Lemma

Impossible to have a graph on an odd number of verts where every vertex is of odd degree.
And NOW to our improvements on small Ramsey numbers.
$R(3, 4) \leq 9$ Case 1

Assume we have a 2-coloring of the edges of $K_9$. 
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1) If any of $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$ are RED, have RED $K_3$. 

2) If all of $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$ are BLUE, have BLUE $K_4$. 

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Case 2 \((\exists v)[\deg_R(v) \leq 2]\), so \(\deg_B(v) \geq 6\).
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(1) There is a RED \(K_3\) in \(\{1, 2, 3, 4, 5, 6\}\). Have RED \(K_3\).

(2) There is a BLUE \(K_3\). With \(v\) get a BLUE \(K_4\).
Recall

Case 1: \( \exists v \left[ \deg R(v) \geq 4 \right] \).
Case 2: \( \exists v \left[ \deg R(v) \leq 2 \right] \).

Negation of Case 1 and Case 2 yields

Case 3: \( \forall v \left[ \deg R(v) = 3 \right] \).

So the RED graph is a graph on 9 verts with all verts of degree 3. This is impossible!
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**Theorem** $R(a, b) \leq$

1. $R(a, b - 1) + R(a - 1, b)$ always.
2. $R(a, b - 1) + R(a - 1, b) - 1$ if $R(a, b - 1) \equiv R(a - 1, b) \equiv 0 \pmod{2}$
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Proof left to the Reader.
Some Better Upper Bounds

- $R(3, 3) \leq R(2, 3) + R(3, 2) \leq 3 + 3 = 6$.  
- $R(3, 4) \leq R(2, 4) + R(3, 3) \leq 4 + 6 - 1 = 9$.  
- $R(3, 5) \leq R(2, 5) + R(3, 4) \leq 5 + 9 = 14$.  
- $R(3, 6) \leq R(2, 6) + R(3, 5) \leq 6 + 14 - 1 = 19$.  
- $R(3, 7) \leq R(2, 7) + R(3, 6) \leq 7 + 19 = 26$.  
- $R(4, 4) \leq R(3, 4) + R(4, 3) \leq 9 + 9 = 18$.  
- $R(4, 5) \leq R(3, 5) + R(4, 4) \leq 14 + 18 - 1 = 31$.  
- $R(5, 5) \leq R(4, 5) + R(5, 4) = 62$. 

Are these tight?
$R(3, 3) \geq 6$

$R(3, 3) \geq 6$: Need coloring of $K_5$ w/o mono $K_3$. 

Note $-1 = 2^2 \pmod{5}$. Hence $a - b \in \mathbb{SQ}$ iff $b - a \in \mathbb{SQ}$. So the coloring is well defined.
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Vertices are $\{0, 1, 2, 3, 4\}$. 

$\text{COL}(a, b) = \text{RED}$ if $a - b \equiv SQ \pmod{5}$, otherwise $\text{BLUE}$.

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\[ \text{COL}(a, b) = \text{RED} \text{ if } a - b \equiv SQ \pmod{5}, \text{ BLUE OW.} \]

- Squares mod 5: 1, 4.
- If there is a RED triangle then \( a - b, b - c, c - a \) all SQ’s. SUM is 0. So

\[ x^2 + y^2 + z^2 \equiv 0 \pmod{5} \text{ Can show impossible} \]

- If there is a BLUE triangle then \( a - b, b - c, c - a \) all non-SQ’s. Product of non-sq’s is a sq. So

\[ 2(a - b), 2(b - c), 2(c - a) \] all squares. SUM to zero- same proof.

**UPSHOT** \( R(3, 3) = 6 \) and the coloring used math of interest!
\( R(4, 4) = 18 \)

\( R(4, 4) \geq 18 \): Need coloring of \( K_{17} \) w/o mono \( K_4 \).
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Use
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Same idea as above for \( K_5 \), but more cases.

\textbf{UPSHOT} \( R(4, 4) = 18 \) and the coloring used math of interest!
\( R(3, 5) = 14 \)

\( R(3, 5) \geq 14 \): Need coloring of \( K_{13} \) w/o RED \( K_3 \) or BLUE \( K_5 \).
\( R(3, 5) = 14 \)

\[ R(3, 5) \geq 14: \text{ Need coloring of } K_{13} \text{ w/o RED } K_3 \text{ or BLUE } K_5. \]

Vertices are \{0, \ldots, 13\}.

Use

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$R(3, 4) = 9$

This is a subgraph of the $R(3, 5)$ graph
$R(3, 4) = 9$

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**UPSHOT**  $R(3, 4) = 9$ and the coloring used math of interest!
Can we extend these Patterns?

**Good news** \( R(4, 5) = 25. \)
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Good news $R(4, 5) = 25$.

Bad news
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**Bad news**
THATS IT.
Can we extend these Patterns?

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**Bad news**

THATS IT.

No other $R(a, b)$ are known using NICE methods.
## Summary of Bounds

<table>
<thead>
<tr>
<th>( R(a, b) )</th>
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<th>New Bound</th>
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$R(5, 5)$: $43 \leq R(5, 5) \leq 49$. So far not mathematically interesting.
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Moral of the Story

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1. At first there seemed to be interesting mathematics with mods and primes leading to nice graphs.
   (Joel Spencer) The Law of Small Numbers: Patterns that persist for small numbers will vanish when the calculations get to hard.

2. Seemed like a nice Math problem that would involve interesting and perhaps deep mathematics. No. The work on it is interesting and clever, but (1) the math is not deep, and (2) progress is slow.
When Will We Know $R(5, 5)$

1. (Quote from Joel Spencer): Erdos asks us to imagine an alien force, vastly more powerful than us, landing on Earth and demanding the value of $R(5, 5)$ or they will destroy our planet. In that case, he claims, we should marshal all our computers and all our mathematicians and attempt to find the value. But suppose, instead, that they ask for $R(6, 6)$. In that case, he believes, we should attempt to destroy the aliens.
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