250H – Discussion Statements

Here are some of the statements and definitions I went over in discussion, as well as some more useful ones you may want to think about:

- The set of even integers

$$\mathbb{Z}_e = \{x \in \mathbb{Z} : \exists k \in \mathbb{Z}, x = 2k\}$$

- The sum of two even integers is an even integer:

$$\forall x, y \in \mathbb{Z}_e \; \exists k \in \mathbb{Z} : x + y = 2k$$

- The set of all positive integers:

$$\mathbb{Z}^+ = \{x \in \mathbb{Z} : x > 0\}$$

- The sum of two positive integers is positive

$$\forall x, y \in \mathbb{Z}^+, x + y \in \mathbb{Z}^+$$

or

$$x, y \in \mathbb{Z}^+ \implies x + y > 0$$

- Every real number, except 0, has a multiplicative inverse

$$\forall x \in \mathbb{R} \setminus \{0\} \; \exists y \in \mathbb{R} \setminus \{0\} : x \cdot y = 1$$

- The set of Natural Numbers (starting at 1 for this example) is infinite

$$\forall n \in \mathbb{N} \; \exists m \in \mathbb{N} : m > n$$

or

$$1 \in \mathbb{N} \wedge \forall n \in \mathbb{N} : n + 1 \in \mathbb{N}$$

- A function $f : X \to Y$ is one-to-one (injective) if

$$\forall x_1, x_2 \in X, f(x_1) = f(x_2) \implies x_1 = x_2$$

or

$$\forall x_1, x_2 \in X \wedge x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$$

- A function $f : X \to Y$ is onto (surjective)

$$\forall y \in Y \; \exists x \in X : f(x) = y$$

- A function is bijective if it is both one-to-one and onto
- The cardinality of a set S, denoted $|S|$, is the number of elements in S.
- The set of prime numbers

$$\mathbb{P} = \{p \in \mathbb{Z}^+ : p > 1 \land \forall a, b \in \mathbb{Z}^+ \land a \cdot b = p \implies (a = 1 \lor b = 1)\}$$

- The set of composite numbers

$$C = \{x \in \mathbb{Z}^+ : \exists z, y \in \mathbb{Z}^+ \land z, y \neq 1 \land x = z \cdot y\}$$

- The set of odd integers

$$\mathbb{Z}_o = \{x \in \mathbb{Z} : \exists k \in \mathbb{Z}, x = 2k + 1\}$$

- All sufficiently large odd numbers are sums of three primes

$$\exists n \in \mathbb{N} \; \forall x \in \mathbb{Z}_o^+ (x > n \implies \exists p_1, p_2, p_3 \in \mathbb{P}, x = p_1 + p_2 + p_3)$$