BILL, RECORD LECTURE!!!!

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Public Key Crypto: Math Needed and Diffie-Hellman

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Yes! And that is the **key** to public-**key** cryptography.

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3. We use hardness assumptions (e.g. factoring is hard).

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What Counts We count math operations as taking 1 step. This could be an issue with enormous numbers. We will work with mods so not a problem.

Math Needed for Both Diffie-Hellman and RSA

Notation

Let p be a prime.

1. $\mathbb{Z}_p$ is the numbers $\{0, \ldots, p-1\}$ with mod add and mult.
2. $\mathbb{Z}_p^*$ is the numbers $\{1, \ldots, p-1\}$ with mod mult.

Convention By **prime** we will always mean a large prime, so in particular, NOT 2. Hence we can assume $\frac{p-1}{2}$ is in $\mathbb{N}$.

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Generators and Discrete Logarithms

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3 is a **generator** for $\mathbb{Z}_7^*$.
Definition: If p is a prime and $\{g^1, \ldots, g^{p-1}\} = \{1, \ldots, p-1\}$ then g is a **generator** for $\mathbb{Z}_p^*$.

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Fact: 3 is a generator mod 101. All math is mod 101.
Discuss the following with your neighbor:

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$3^x \equiv 92$ easy. $3^x \equiv 93$ Not known how hard.

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2. So finding x such that $g^x \equiv p - g^a \equiv -g^a$ is as easy as g^a.

$$x = \frac{p-1}{2} + a : \qquad g^{\frac{p-1}{2}+a} = g^{\frac{p-1}{2}} g^a \equiv -g^a$$

Discrete Log-Example: $3^x \equiv 93 \pmod{101}$

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Is there a trick for $g^x \equiv 93 \pmod{101}$? Not that I know of.

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- A **good** alg would be time $(\log p)^{O(1)}$.
- A **bad** alg would be time $p^{O(1)}$.
- If an algorithm is in time (say) $p^{1/10}$ still not efficient but will force Alice and Bob to up their game.

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Good Candidate for a hard problem for Eve.

Bill's Opinion on DL. Also Applies to Factoring

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   It won't happen to me Until it does.

Discrete Log-General

Definition Let p be a prime and g be a generator mod p.
The **Discrete Log Problem:**
Given $a \in \{1, \ldots, p\}$, find x such that $g^x \equiv a \pmod{p}$. We call this $DL_{p,g}(a)$.

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4. **Tradeoff:** By restricting a we are cutting down search space for Eve. Even so, in this case we need to since she REALLY can recognize when DL is easy.

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Do we have this?

No. But we'll come close.

Convention

For the rest of the slides on **Diffie-Hellman Key Exchange** there will always be a prime p that we are considering.

ALL math done from that point on is mod p.

ALL numbers are in $\{1, \ldots, p-1\}$.

Finding Generators

Finding Gens; How Many Gens Are There?

Problem Given p, find g such that

- g generates $\mathbb{Z}_p^*$.
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Hence if you just look for a gen you will find one soon.

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Given prime p, find a gen for $\mathbb{Z}_p^*$

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Bad! Recall $(\log p)^{O(1)}$ is fast, $O(p)$ is slow.

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CAVEAT We need to pick certain kinds of primes. **Can** do that!

BILL, STOP RECORDING LECTURE!!!!

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