BILL TAPE LECTURE

Diffie-Helman Key Exchange

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Convention (Possibly Repeated)

For the rest of the slides on **Diffie-Hellman Key Exchange** there will always be a prime p that we are considering and a generator $g \in \{\frac{p}{3}, \frac{2p}{3}\}$. We omit the bounds on g.

ALL arithmetic done from that point on is mod p.

ALL numbers are in $\{1, \ldots, p-1\}$.

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Biggest PRO: Alice and Bob never had to meet!
Question: Can Eve find out s?

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10. At the count of 3 both yell out your number at the same time.

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Question: If Eve can crack DH then Eve can compute ???.

Hardness Assumption

Definition Let DHF be the following function:
Inputs: p, g, g^a, g^b (note that a, b are not the input)

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Obvious Theorem: If Alice can crack Diffie-Hellman quickly then Alice can compute DHF quickly.

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1. Nobody has found a way to solve DHF quickly that does not involve solving Discrete Log.
2. Discrete Log is believed to be hard.
3. Still, would be nice to have a key exchange based on DL.

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Next Slide continues this discussion.

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At the end Alice and Bob have s but **s has no meaning!**.
s is not going to be **The Matrix.**
s is going to be some random number in $\{1, \ldots, p-1\}$.

How can Alice and Bob Use s?

s is random.

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s is random. No meaning.

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When life gives you a lemon, make lemonade.

How can Alice and Bob Use s?

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When life gives you a lemon, make lemonade.

There are ciphers that use a random string as their key.
(The 1-time pad is such a cipher.)

Misc Points about DH Key Exchange?

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4. DHF proven to be hard. (IMHO not gonna happen.)

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4. Eve could measure how much time it takes for Bob to know the string and use that to narrow down the space of strings.

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3. Eve lets Bob send g^b without interference.
4. Alice thinks the shared secret string is g^{ab}.
   Bob thinks the shared secret string is $g^{a'b}$.

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What if Eve could intercept both messages and replace them.

1. Alice sends g^a.
2. Eve intercepts the message, picks a random a', and instead sends on to Bob $g^{a'}$.
3. Eve lets Bob send g^b without interference.
4. Alice thinks the shared secret string is g^{ab}.
   Bob thinks the shared secret string is $g^{a'b}$.
   So Alice and Bob will not be able to communicate, which is a win for Eve.

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Example: Elliptic Curve Diffie-Hellman (actually used).
Example: Braid Diffie-Hellman (not actually used).

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Sounds like DH is vulnerable! I posted about this on my blog and got responses (next slide).

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5. Jon Katz asked them for their code. They declined.

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   2.3 If you publish an **academic paper** about cracking DL, you should have the code and make it available. See next point.
   2.4 If you actually worry about DH being cracked then tell the crypto companies or the government first. (See the fiction book **Factorman**. I reviewed it: `https://www.cs.umd.edu/users/gasarch/BLOGPAPERS/factorman.pdf`)

BILL, STOP RECORDING LECTURE!!!!

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