Solution to Hw 03 Problems 4 and 5

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1. Give an algorithm that does the following.
   **Input:** \( \alpha \) a regular expression
   **Output:** \( \beta \) a regular expression for the complement of the language represented by \( \alpha \)

   You may use any procedure described in class (e.g. procedures for intersection, union, complementation, powerset construction, equivalences between NFAs and DFAs, conversion from NFA to regular expression).

2. Think about: if \( \alpha \) is of length \( n \), how long is \( \beta \)?
SOLUTION
1. Input \( \alpha \) (\(|\alpha|\) is of length \( n \))
2. Convert \textit{alpha} to NFA \( N \) (\( N \) has \( O(n) \) states)
3. Convert \( N \) to DFA \( M \) using powerset construction (\( M \) has \( O(2^n) \) states)
4. Swap final and nonfinal states of \( M \) to get \( M' \).
5. Convert \( M' \) to a regular expression \( \beta \) via \( R(i, j, k) \) method (\(|\beta|\) is \( O(2^{2^n}) \))

END OF SOLUTION
Let $L = \{w : \#_a(w) \equiv 0 \pmod{2}\}$ and the alphabet is $\{a, b\}$.

1. Draw the DFA that accepts this language.

2. Use the $R(i, j, k)$ method to find the regex for this language. YOU MUST USE THIS METHOD EXACTLY AS GIVEN.

3. Determine the regex for $L$ WITHOUT using the $R(i, j, k)$ method. It must be a SIMPLE regex. If we cannot understand your regex, YOU WILL GET NO CREDIT.
The following is the DFA for \( L = \{ w : \#_a(w) \equiv 0 \pmod{2} \} \).
We begin with

\[ \bigcup_{f \in F} R(1, f, n) = R(1, 1, 2) = R(1, 1, 1) \cup R(1, 2, 1) R(2, 2, 1) R(2, 1, 1) \]

so we ONLY want \( R(1, 1, 2) \). We have

\[ R(1, 1, 0) = \{ \{ e \} \} \cup \{ b \} = \{ \{ e \}, b \} \]

\[ R(1, 2, 0) = \{ a \} \]

\[ R(2, 1, 0) = \{ a \} \]

\[ R(2, 2, 0) = \{ \{ e \} \} \cup \{ b \} = \{ \{ e \}, b \} \]
$R(1, 1, 1) = R(1, 1, 0) \cup R(1, 1, 0)R(1, 1, 0)^*R(1, 1, 0) =$

$
\{\{e\}, b\} \cup \{\{e\}, b\}\{\{e\}, b\}^*\{\{e\}, b\} = b^*
$
\( R(1, 1, 1) = R(1, 1, 0) \cup R(1, 1, 0)R(1, 1, 0)^* R(1, 1, 0) = \)
\[ \{\{e\}, b\} \cup \{\{e\}, b\}\{\{e\}, b\}^* \{\{e\}, b\} = b^* \]

\( R(1, 2, 1) = R(1, 2, 0) \cup R(1, 1, 0)R(1, 1, 0)^* R(1, 2, 0) = \)
\[ \{a\} \cup \{\{e\}, b\}\{\{e\}, b\}^* a = b^* a \]
\[ R(1, 1, 1) = R(1, 1, 0) \cup R(1, 1, 0)R(1, 1, 0)^* R(1, 1, 0) = \]
\[ \{\{e\}, b\} \cup \{\{e\}, b\}\{\{e\}, b\}\{\{e\}, b\} = b^* \]

\[ R(1, 2, 1) = R(1, 2, 0) \cup R(1, 1, 0)R(1, 1, 0)^* R(1, 2, 0) = \]
\[ \{a\} \cup \{\{e\}, b\}\{\{e\}, b\}^* a = b^* a \]

\[ R(2, 2, 1) = R(2, 2, 0) \cup R(2, 1, 0)R(1, 1, 0)^* R(1, 2, 0) = \]
\[ \{\{e\}, b\} \cup a\{\{e\}, b\}^* a = \{e, b\} \cup ab^* a \]
\( R(1,1,1) = R(1,1,0) \cup R(1,1,0)R(1,1,0)^*R(1,1,0) = \{\{e\}, b\} \cup \{\{e\}, b\}\{\{e\}, b\}^*\{\{e\}, b\} = b^* \)

\( R(1,2,1) = R(1,2,0) \cup R(1,1,0)R(1,1,0)^*R(1,2,0) = \{a\} \cup \{\{e\}, b\}\{\{e\}, b\}^*a = b^*a \)

\( R(2,2,1) = R(2,2,0) \cup R(2,1,0)R(1,1,0)^*R(1,2,0) = \{\{e\}, b\} \cup a\{\{e\}, b\}^*a = \{e, b\} \cup ab^*a \)

\( R(2,1,1) = R(2,1,0) \cup R(2,1,0)R(1,1,0)^*R(1,1,0) = \{a\} \cup a\{\{e\}, b\}^*\{\{e\}, b\} = ab^* \)

All we need is \( R(1,1,2) \). So our final regex is \( b^* \cup b^*a(\{\{e\}, b\} \cup ab^*a)^*ab^*a \).
\[ R(1, 1, 1) = R(1, 1, 0) \cup R(1, 1, 0)R(1, 1, 0)^*R(1, 1, 0) = \\
\{\{e\}, b\} \cup \{\{e\}, b\}\{\{e\}, b\}^*\{\{e\}, b\} = b^* \]

\[ R(1, 2, 1) = R(1, 2, 0) \cup R(1, 1, 0)R(1, 1, 0)^*R(1, 2, 0) = \\
\{a\} \cup \{\{e\}, b\}\{\{e\}, b\}^*a = b^*a \]

\[ R(2, 2, 1) = R(2, 2, 0) \cup R(2, 1, 0)R(1, 1, 0)^*R(1, 2, 0) = \\
\{\{e\}, b\} \cup a\{\{e\}, b\}^*a = \{e, b\} \cup ab^*a \]

\[ R(2, 1, 1) = R(2, 1, 0) \cup R(2, 1, 0)R(1, 1, 0)^*R(1, 1, 0) = \\
\{a\} \cup a\{\{e\}, b\}^*\{\{e\}, b\} = ab^* \]

All we need is \( R(1, 1, 2) \). So our final regex is

\[ b^* \cup b^*a(\{\{e\}, b\} \cup ab^*a)^*ab^* \]