BILL, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!
Tricks for Divisibility and DFA’s
For this Slide Packet $\Sigma = \{0, \ldots, 9\}$
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Strings are numbers in base 10. The string $d_n \cdots d_0$ is the number $d_n \times 10^n + \cdots + d_1 \times 10^1 + d_0 \times 10^0$. 
For this Slide Packet $\Sigma = \{0, \ldots, 9\}$.

Strings are numbers in base 10. The string

$$d_n \cdots d_0$$

is the number

$$d_n \times 10^n + \cdots + d_1 \times 10^1 + d_0 \times 10^0.$$ 

We feed a number into a DFA right-to-left: $d_0$, then $d_1$ etc.
Trick for Mod 2. $\equiv$ is Mod 2

Did you know? $n \equiv 0$ iff its last digit is $\equiv 0$. 
Trick for Mod 2. ≡ is Mod 2

Did you know? \( n \equiv 0 \) iff its last digit is \( \equiv 0 \).
We state this a different way so can generalize later.
Trick for Mod 2. \( \equiv \) is Mod 2

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Thm \( d_n \cdots d_0 \equiv d_0 \pmod{2} \).
Did you know? $n \equiv 0$ iff its last digit is $\equiv 0$.
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**Thm** \(d_n \cdots d_0 \equiv d_0 \pmod{2} \).

**Pf**

\[
d_n \times 10^n + \cdots + d_1 \times 10 + d_0 = 10(d_n \times 10^{n-1} + \cdots + d_1) + d_0 \equiv d_0.
\]
DFA for Mod 2
DFA for Mod 2
Trick for Mod 3. \( \equiv \) is Mod 3

Did you Know? \( d \equiv 0 \) iff sum of digits is \( \equiv 0 \).
Trick for Mod 3. ≡ is Mod 3

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**Thm** \( d_n \cdots d_0 \equiv d_n + \cdots + d_0 \).
Did you Know? \( d \equiv 0 \) iff sum of digits is \( \equiv 0 \).

We state this a different way which gives more information.

**Thm** \( d_n \cdots d_0 \equiv d_n + \cdots + d_0 \).

**Pf** I’ll have 250H prove by induction: \( (\forall n)[10^n \equiv 1] \). Hence
Trick for Mod 3. \( \equiv \) is Mod 3

Did you Know? \( d \equiv 0 \) iff sum of digits is \( \equiv 0 \).
We state this a different way which gives more information.

**Thm** \( d_n \cdots d_0 \equiv d_n + \cdots + d_0 \).

**Pf** I’ll have 250H prove by induction: \((\forall n)[10^n \equiv 1]\). Hence

\[
d_n \times 10^n + \cdots + d_1 \times 10 + d_0 \times 10^0
\]
Trick for Mod 3. $\equiv$ is Mod 3

**Did you Know?** $d \equiv 0$ iff sum of digits is $\equiv 0$.

We state this a different way which gives more information.

**Thm** $d_n \cdots d_0 \equiv d_n + \cdots + d_0$.

**Pf** I’ll have 250H prove by induction: $(\forall n)[10^n \equiv 1]$. Hence

$$d_n \times 10^n + \cdots + d_1 \times 10 + d_0 \times 10^0$$

$$\equiv d_n \times 1 + \cdots + d_1 \times 1 + d_0 \times 1$$
Trick for Mod 3. \( \equiv \) is Mod 3

Did you Know? \( d \equiv 0 \) iff sum of digits is \( \equiv 0 \).

We state this a different way which gives more information.

**Thm** \( d_n \cdots d_0 \equiv d_n + \cdots + d_0 \).

**Pf** I’ll have 250H prove by induction: \((\forall n)[10^n \equiv 1]\). Hence

\[
d_n \times 10^n + \cdots + d_1 \times 10 + d_0 \times 10^0
\]

\[
\equiv d_n \times 1 + \cdots + d_1 \times 1 + d_0 \times 1
\]

\[
\equiv d_n + \cdots + d_1 + d_0
\]
DFA for Divisible by 3
DFA for Divisible by 3
Trick for Mod 4. All ≡ are Mod 4

Did you Know?  \( n \equiv 0 \) iff
Trick for Mod 4. All $\equiv 0$ are Mod 4

Did you Know? $n \equiv 0$ iff last 2 digits are a number $\equiv 0$. 
Trick for Mod 4. All \( \equiv \) are Mod 4

Did you Know? \( n \equiv 0 \) iff last 2 digits are a number \( \equiv 0 \).

Thm \( d_n \cdots d_0 \equiv 2d_1 + d_0 \).
Trick for Mod 4. All $\equiv 0$ are Mod 4

Did you Know? $n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_n \cdots d_0 \equiv 2d_1 + d_0$.

Pf I’ll have 250H prove by induction ($\forall n \geq 2) [10^n \equiv 0]$. Hence
Trick for Mod 4. All $\equiv 0$ are Mod 4

Did you Know? $n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_n \cdots d_0 \equiv 2d_1 + d_0$.

Pf I’ll have 250H prove by induction $(\forall n \geq 2)[10^n \equiv 0]$. Hence

$$d_n \times 10^n + \cdots + d_1 \times 10 + d_0$$
Did you Know? \( n \equiv 0 \) iff last 2 digits are a number \( \equiv 0 \).

Thm \( d_n \cdots d_0 \equiv 2d_1 + d_0 \).

Pf I’ll have 250H prove by induction (\( \forall n \geq 2 \))[10^n \equiv 0]. Hence

\[
d_n \times 10^n + \cdots + d_1 \times 10 + d_0
\]

\begin{align*}
&\equiv d_1 \times 10 + d_0
\end{align*}
Trick for Mod 4. All $\equiv$ are Mod 4

Did you Know? $n \equiv 0$ iff last 2 digits are a number $\equiv 0$.

Thm $d_n \cdots d_0 \equiv 2d_1 + d_0$.

Pf I’ll have 250H prove by induction $(\forall n \geq 2)[10^n \equiv 0]$. Hence

$$d_n \times 10^n + \cdots + d_1 \times 10 + d_0$$

$\equiv d_1 \times 10 + d_0$

$\equiv 2d_1 + d_0$. 
DFA for Divisible by 4
DFA for Divisible by 4

The diagram represents a Deterministic Finite Automaton (DFA) designed to recognize strings divisible by 4. The states are labeled with numbers that indicate the values of digits in the input string that lead to an accepting state. The transitions are labeled with digits that, when added to the current state, result in an accepting state. The accepting state is marked with a double circle.
Key to all of these Problems

For all of these problems we need to find a pattern of $10^n \pmod{a}$. 
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For all of these problems we need to find a pattern of $10^n \pmod{a}$. Mod 2: Pattern is $1,0,0,0,\ldots$, DFA only cared about first digit.
For all of these problems we need to find a pattern of $10^n \pmod{a}$. Mod 2: Pattern is 1,0,0,0,..., DFA only cared about first digit. Mod 3: Pattern is 1,1,1,1,..., DFA tracked sum of digits mod 3.
For all of these problems we need to find a pattern of $10^n \pmod{a}$. 
Mod 2: Pattern is 1,0,0,0,..., DFA only cared about first digit. 
Mod 3: Pattern is 1,1,1,1,..., DFA tracked sum of digits mod 3. 
Mod 4: Pattern is 1,2,0,0,0,..., DFA only cared about first 2 digits.
Tricks for Mod 5 and Mod 6

These may be on a HW.
Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?

**Did you Know?** $n \equiv 0$ iff $\pm$ sum of digits is $\equiv 0$. 
Is there a trick for mod 11?

**Did you Know?** $n \equiv 0$ iff $\pm$ sum of digits is $\equiv 0$.

**Thm** $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots d_n$. 
Trick for Mod 11. All $\equiv$ are Mod 11

Is there a trick for mod 11?

Did you Know? $n \equiv 0$ iff $\pm$ sum of digits is $\equiv 0$.

Thm $d_n \cdots d_0 \equiv d_0 - d_1 + d_2 - \cdots d_n$.

Proof may be on HW or Midterm or Final or some combination.
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]
DFA for Mod 11

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\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]

\[ s = (0, 0). \]
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Final state: Not going to have these, this is DFA-classifier.
DFA for Mod 11

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\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]

\[ s = (0, 0). \]

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\[
\delta((i,j),\sigma) \begin{cases} 
(i + \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 0 \\
(i - \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 1 
\end{cases}
\] (1)
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]

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\[ \delta((i, j), \sigma) \begin{cases} (i + \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 0 \\ (i - \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 1 \end{cases} \tag{1} \]

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]

\[ s = (0, 0) \]

Final state: Not going to have these, this is DFA-classifier.

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\delta((i, j), \sigma) = \begin{cases} 
(i + \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 0 \\
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\end{cases}
\]  

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

22 states.
DFA for Mod 11

Need to keep track of both the running weighted sum mod 11 and if you are reading an even or odd place.

\[ Q = \{0, \ldots, 10\} \times \{0, 1\} \]

\[ s = (0, 0). \]

Final state: Not going to have these, this is DFA-classifier.

\[
\delta((i,j), \sigma) = \begin{cases} 
(i + \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 0 \\
(i - \sigma \pmod{11}, j + 1 \pmod{2}) & \text{if } j = 1 
\end{cases}
\] 

(1)

We keep track of a running weighted sum mod 11 and position of the symbol mod 2.

22 states.

**Classifier** If end in \((i, 0)\) or \((i, 1)\) then number is \(\equiv i\).
Is there a trick for mod 7? VOTE

Pattern is 1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, ...
Is there a trick for mod 7? VOTE

Answer Depends what you call a trick.
Is There a Mod 7 Trick? \( \equiv \) is Mod 7

Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick.
We need to spot a pattern.
Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick.

We need to spot a pattern.

$10^0 \equiv 1$
Is there a trick for mod 7? VOTE

*Answer* Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$

$10^1 \equiv 3$
Is There a Mod 7 Trick? ≡ is Mod 7

Is there a trick for mod 7? VOTE

Answer Depends what you call a trick.
We need to spot a pattern.

$10^0 \equiv 1$
$10^1 \equiv 3$
$10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$
Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick.

We need to spot a pattern.

- $10^0 \equiv 1$
- $10^1 \equiv 3$
- $10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2$
- $10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6$
Is there a trick for mod 7? VOTE

Answer Depends what you call a trick.
We need to spot a pattern.

\[10^0 \equiv 1\]
\[10^1 \equiv 3\]
\[10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2\]
\[10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6\]
\[10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4\]
Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick.
We need to spot a pattern.

\[10^0 \equiv 1\]
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\[10^2 \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2\]
\[10^3 \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6\]
\[10^4 \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4\]
\[10^5 \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5\]
Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick.

We need to spot a pattern.

\[
\begin{align*}
10^0 & \equiv 1 \\
10^1 & \equiv 3 \\
10^2 & \equiv 10 \times 10 \equiv 3 \times 3 \equiv 9 \equiv 2 \\
10^3 & \equiv 10^2 \times 10 \equiv 2 \times 3 \equiv 6 \\
10^4 & \equiv 10^3 \times 10 \equiv 6 \times 3 \equiv 18 \equiv 4 \\
10^5 & \equiv 10^4 \times 10 \equiv 4 \times 3 \equiv 12 \equiv 5 \\
10^6 & \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1
\end{align*}
\]
Is There a Mod 7 Trick? \( \equiv \) is Mod 7

Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick. We need to spot a pattern.

\[ 10^0 \equiv 1 \]
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\[ 10^6 \equiv 10^5 \times 10 \equiv 5 \times 3 \equiv 15 \equiv 1 \]

Pattern is 1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, . . .
Is there a trick for mod 7? VOTE

**Answer** Depends what you call a trick.

We need to spot a pattern.

\[ 10^0 \equiv 1 \]
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Pattern is 1, 3, 2, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, . . . .

Can we use this?
Using the Divide by 7 Trick

Want to know what $3876554$ is mod 7. All arith is mod 7.

$4 \times 1 + 5 \times 3 + 5 \times 2 + 6 \times 6 + 7 \times 4 + 8 \times 5 + 3 \times 1$

We do this mod 7 so the numbers do not get that big

$4 + 15 + 10 + 36 + 28 + 40 + 3\equiv (4+3+1)+(3+1+5+3)\equiv 1+5\equiv 6$. 

DFA States will keep track of Running weighted sum mod 7 Position of digit mod 6 so know which weights to use. So there are $7 \times 6 = 42$ states.
Using the Divide by 7 Trick

Want to know what $3876554$ is mod 7. All arith is mod 7. 
$4 \times 1 + 5 \times 3 + 5 \times 2 + 6 \times 6 + 7 \times 4 + 8 \times 5 + 3 \times 1$

We do this mod 7 so the numbers do not get that big

$$4 + 15 + 10 + 36 + 28 + 40 + 3$$

$$\equiv 4 + 1 + 3 + 1 + 0 + 5 + 3 \equiv (4 + 3 + 1) + (3 + 1 + 5 + 3) \equiv 1 + 5 \equiv 6.$$
Using the Divide by 7 Trick

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$$4 \times 1 + 5 \times 3 + 5 \times 2 + 6 \times 6 + 7 \times 4 + 8 \times 5 + 3 \times 1$$

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**DFA** States will keep track of
Using the Divide by 7 Trick

Want to know what $3876554$ is mod $7$. All arith is mod $7$.

$$4 \times 1 + 5 \times 3 + 5 \times 2 + 6 \times 6 + 7 \times 4 + 8 \times 5 + 3 \times 1$$

We do this mod $7$ so the numbers do not get that big

$$4 + 15 + 10 + 36 + 28 + 40 + 3$$

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**DFA** States will keep track of

Running weighted sum mod 7
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7. All arith is mod 7.
4 × 1 + 5 × 3 + 5 × 2 + 6 × 6 + 7 × 4 + 8 × 5 + 3 × 1

We do this mod 7 so the numbers do not get that big

\[ 4 + 15 + 10 + 36 + 28 + 40 + 3 \]

\[ \equiv 4 + 1 + 3 + 1 + 0 + 5 + 3 \equiv (4 + 3 + 1) + (3 + 1 + 5 + 3) \equiv 1 + 5 \equiv 6. \]

**DFA** States will keep track of

Running weighted sum mod 7

Position of digit mod 6 so know which weights to use.
Using the Divide by 7 Trick

Want to know what 3876554 is mod 7. All arith is mod 7.

\[ 4 \times 1 + 5 \times 3 + 5 \times 2 + 6 \times 6 + 7 \times 4 + 8 \times 5 + 3 \times 1 \]

We do this mod 7 so the numbers do not get that big

\[ 4 + 15 + 10 + 36 + 28 + 40 + 3 \]

\[ \equiv 4 + 1 + 3 + 1 + 0 + 5 + 3 \equiv (4 + 3 + 1) + (3 + 1 + 5 + 3) \equiv 1 + 5 \equiv 6. \]

**DFA** States will keep track of
Running weighted sum mod 7
Position of digit mod 6 so know which weights to use.
So there are \( 7 \times 6 = 42 \) states.
Is the Method a Trick?

YES

A DFA can do it.

NO

A human can't do it easily— the pattern is not like 1, 1, 1, ...
or mostly 0's.
Is the Method a Trick?

**YES**  A DFA can do it.
Is the Method a Trick?

YES A DFA can do it.
NO A human can’t do it easily- the pattern is not like 1,1,1,... or mostly 0’s.
The DFA for \( \{ n : n \equiv 0 \ (\text{mod} \ 7) \} \)
The DFA for $\{ n : n \equiv 0 \pmod{7} \}$

**BILL:** Saadiq, please draw a DFA for $\{ n : n \equiv 0 \pmod{7} \}$. 
The DFA for \( \{ n : n \equiv 0 \pmod{7} \} \)

**BILL:** Saadiq, please draw a DFA for \( \{ n : n \equiv 0 \pmod{7} \} \).

**SAADIQ:** No. Ask Yaelle.
The DFA for \( \{ n : n \equiv 0 \pmod{7} \} \)

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The DFA for \( \{ n : n \equiv 0 \ (\text{mod} \ 7) \} \)

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::
The DFA for \( \{ n : n \equiv 0 \pmod{7} \} \)

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BILL: Saadiq, please draw a DFA for \( \{ n : n \equiv 0 \pmod{7} \} \).
SAADIQ: No. Ask Yaelle.

Might make it a HW to do as a table.
Possible Research Question

What is the fastest way to determine $n \pmod{7}$?
Possible Research Question

What is the fastest way to determine \( n \) (mod 7)?

**Method One** Divide and take remainder.
Possible Research Question

What is the fastest way to determine \( n \pmod{7} \)?

**Method One** Divide and take remainder.

**Method Two** Use the DFA.
What is the fastest way to determine $n \pmod{7}$?

**Method One** Divide and take remainder.

**Method Two** Use the DFA.

**Question** Which is faster?
Possible Research Question

What is the fastest way to determine $n \pmod{7}$?

**Method One** Divide and take remainder.

**Method Two** Use the DFA.

**Question** Which is faster?

Might be hard to tell because today’s computers are so fast!
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