A Tale of Two Books

William Gasarch

November 21, 2019
Problems with a Point: Exploring Math and Computer Science

William Gasarch
Clyde Kruskal
Ever notice how civilians (that is non-math people) use math words badly? Ever notice how sometimes you know a math statement is false (or not known) since if it was true you would know it?

Each chapter of this book makes a point like those above and then illustrates the point by doing some real mathematics.

This book gives readers valuable information about how mathematics and theoretical computer science work, while teaching them some actual mathematics and computer science through examples and exercises. Much of the mathematics could be understood by a bright high school student. The points made can be understood by anyone with an interest in math, from the bright high school student to a Field’s medal winner.
Book’s Origin

- In 2003 Lance Fortnow started **Complexity Blog**
- In 2007 Bill Gasarch joined and it was a co-blog.
- In 2015 various book publishers asked us
  
  Can you make a book out of your blog?

- Lance declined but Bill said **YES**.
Bill took the posts that had the following format:

- make a point **about** mathematics
- do some math to **underscore** those points

and made those into chapters.

**Caveat:** Not every chapter is quite like that.

To quote Ralph Waldo Emerson:

*A foolish consistency is the hobgoblin of small minds.*
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The publisher wisely decided to be less cute and more informative:

Problems with a Point: Exploring Math and Computer Science
After some samples of Bill’s writing the publisher said
Clyde Joins the Project!

After some samples of Bill’s writing the publisher said

Please Procure People to Polish Prose and Proofs of Problems with a Point

so
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Clyde Kruskal became a co-author.
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Now onto some samples of the book!
From the Year 2000 Maryland Math Competition: 
*There are 2000 cans of paint. Show that at least one of the following two statements is true:*

▶ There are at least 45 cans of the same color.
▶ There are at least 45 cans that are different colors.

Work on it.
From the Year 2000 Maryland Math Competition:  
*There are 2000 cans of paint. Show that at least one of the following two statements is true:*

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- There are at least 45 cans that are different colors.

Work on it.  

**Answer:**  
If there are 45 different colors of paint then we are done. Assume there are $\leq 44$ different colors. If all colors appear $\leq 44$ times then there are $44 \times 44 = 1936 < 2000$ cans of paint, a contradiction.  

**Note:** this was Problem 1, which is supposed to be easy and indeed 95% got it right. What about the other 5%? Next slide.
One of the Wrong Answers. Or is it?

There are 2000 cans of paint. Show that at least one of the following two statements is true:

- There are at least 45 cans of the same color.
- There are at least 45 cans that are different colors.

Paint cans are grey. Hence there are all the same color. Therefore there are 2000 cans that are the same color.

What do you think:
- That's just stupid. 0 points.
- Question says cans of the same color... The full 30 pts.
- Not only does he get 30 points, but everyone else should get 0.
There are 2000 cans of paint. Show that at least one of the following two statements is true:

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- There are at least 45 cans that are different colors.

ANSWER:
If you look at a paint color really really carefully there will be differences. Hence, even if two cans seem to both be (say) RED, they are really different. Therefore there are 2000 cans of different colors.

What do you think:

- Thats just stupid. 0 points.
- Well...
  - he's got a point. 30 points in fact.
- Not only does he get 30 points, but everyone else should get 0.
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From the year 2007 Maryland Math Competition.

QUESTION: Let $ABC$ be a fixed triangle. Let $COL$ be any 2-coloring of the plane where each point is colored with red or green. Prove that there is a triangle $DEF$ in the plane such that $DEF$ is similar to $ABC$ and the vertices of $DEF$ all have the same color.
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**Note** I think I was assigned to grade it since it looks like the kind of problem I would make up, even though I didn’t. It was problem 5 (out of 5) and was hard. About 100 students tried it, 8 got full credit, 10 got partial credit.
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Funny Answer One:
All the vertices are red because I can make them whatever color I want. I can also write at a 30 degree angle to the bottom of this paper (The students answer was written at a 30 degree angle to the bottom of the paper.) if thats what I feel like doing at the moment. Just like $2 + 2 = 5$ if thats what my math teacher says. Math is pretty subjective anyway.
Was Student One Serious?

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Theorem

The students is not serious.

Proof

Assume, by contradiction, that they are serious. Then they really think math is subjective. Hence they don't really understand math. Hence they would not have done well enough on Part I to qualify for Part II. But they took Part II. Contradiction.
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I like to think that we live in a world where points are not judged by their color, but by the content of their character. Color should be irrelevant in the plane. To prove that there exists a group of points where only one color is acceptable is a reprehensible act of bigotry and discrimination.
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Was Student Two Serious.
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Was Student Two Serious. Yes.
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Was Student Two Serious. Yes. about **Justice!**.
Bill assigned the following in Discrete Math: For each of the following sequences find a **simple function** $A(n)$ such that the sequence is $A(1), A(2), A(3), \ldots$

1. 10, -17, 24, -31, 38, -45, 52, …
2. -1, 1, 5, 13, 29, 61, 125, …
3. 6, 9, 14, 21, 30, 41, 54, …

**Caveat:** These are NOT trick questions. Work on it.
Point: What is a Simple Function?

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1. 10, -17, 24, -31, 38, -45, 52, ···
   \( A(n) = (-1)^{n+1}(7n + 3). \)
2. -1, 1, 5, 13, 29, 61, 125, ···
   \( A(n) = 2^n - 3. \)
3. 6, 9, 14, 21, 30, 41, 54, ···
   \( A(n) = n^2 + 5. \)

Caveat: These are NOT trick questions.

Work on it.
A Student asks — What is a Simple Function?

One student, in earnest, emailed Bill the following:

*We never defined **Simple Function** in class so I went to Wikipedia. It said that **A Simple Function is a linear combination of indicator functions of measurable sets.** Is that what you want us to use?*
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I doubt Bill knows what those terms mean.

Bill told him NO— all Bill wanted is an easy-to-describe function. Personally I think Bill should have told him to use that definition to see what he came up with.
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Bill told him NO— all Bill wanted is an easy-to-describe function. Personally I think Bill should have told him to use that definition to see what he came up with.
The student got the first one right, but left the other two blank.
Point: When Do Patterns Hold?

The last question brings up the question of when patterns do and don’t hold. We looked for cases where a pattern *did not* hold.
First Non-Pattern: $n$ Points on a circle

What is the max number of regions formed by connecting every pair of $n$ points on a circle. For $n = 1, 2, 3, 4, 5$:

Tempted to guess $2^{n-1}$.
First Non-Pattern: \( n \) Points on a circle

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Tempted to guess $2^{n-1}$.
But for $n = 6$, the number of regions is only 31.
The actual number of regions for $n$ points is $\binom{n}{4} + \binom{n}{2} + 1$. 
Second Non-Pattern: Borwein Integrals

\[
\int_0^\infty \frac{\sin x}{x} = \frac{\pi}{2}
\]

\[
\int_0^\infty \frac{\sin x \sin \frac{x}{3}}{x} = \frac{\pi}{2}
\]

\[
\int_0^\infty \frac{\sin x \sin \frac{x}{3} \sin \frac{x}{5} \sin \frac{x}{7} \sin \frac{x}{9} \sin \frac{x}{11} \sin \frac{x}{13}}{x \ \frac{x}{3} \ \frac{x}{5} \ \frac{x}{7} \ \frac{x}{9} \ \frac{x}{11} \ \frac{x}{13}} = \frac{\pi}{2}
\]
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\[ \int_0^\infty \frac{\sin x \sin \frac{x}{3}}{x \frac{x}{3}} = \frac{\pi}{2} \]

\[ \vdots \]

\[ \int_0^\infty \frac{\sin x \sin \frac{x}{3} \sin \frac{x}{5} \sin \frac{x}{7} \sin \frac{x}{9} \sin \frac{x}{11} \sin \frac{x}{13}}{x \frac{x}{3} \frac{x}{5} \frac{x}{7} \frac{x}{9} \frac{x}{11} \frac{x}{13}} = \frac{\pi}{2} \]

But

\[ \int_0^\infty \frac{\sin x \sin \frac{x}{3} \sin \frac{x}{5} \sin \frac{x}{7} \sin \frac{x}{9} \sin \frac{x}{11} \sin \frac{x}{13} \sin \frac{x}{15}}{x \frac{x}{3} \frac{x}{5} \frac{x}{7} \frac{x}{9} \frac{x}{11} \frac{x}{13} \frac{x}{15}} = \]

\[ \frac{4678079247134407386537864469\pi}{935615849440640907310521750000} \sim 0.99999999999852937186 \times \frac{\pi}{2} \]
Why the breakdown at 15?

Because

\[
\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{13} < 1
\]

but

\[
\frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{15} > 1.
\]

For more Google Borwein Integral
Problems with a Point: Exploring Math and Computer Science
by William Gasarch (Author), Clyde Kruskal (Author)

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Chapter 1 of the sequel

Problems with two Points:
More Explorations of Math and Computer Science

will be

The Mathematics of Book Pricing on Amazon
Mathematical Muffin Morsels: Nobody Wants a Small Piece

William Gasarch
Erik Metz
Jacob Prinz
Daniel Smolyak
Suppose you have five muffins that you want to divide and give to Alice, Bob, and Carol. You want each of them to get 5/3. You could cut each muffin into 1/3-1/3-1/3 and give each student five 1/3-sized pieces. But Alice objects! She has large hands! She wants everyone to have pieces larger than 1/3.

Is there a way to divide five muffins for three students so that everyone gets 5/3, and all pieces are larger than 1/3? Spoiler alert: Yes! In fact, there is a division where the smallest piece is 5/12. Is there a better division? Spoiler alert: No.

In this book we consider THE MUFFIN PROBLEM: what is the best way to divide up m muffins for s students so that everyone gets m/s muffins, with the smallest pieces maximized. We look at both procedures for the problem and proofs that these procedures are optimal.

This problem takes us through much mathematics of interest, for example, combinatorics and optimization theory. However, the math is elementary enough for an advanced high school student.
How I Learned the Muffin Problem

A Recreational Math Conference
(Gathering for Gardner)
May 2016

Bill found a pamphlet:
The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

*How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ where nobody gets a tiny sliver?*
### Five Muffins, Three Students, Proc by Picture

<table>
<thead>
<tr>
<th>Person</th>
<th>Color</th>
<th>What they Get</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>RED</td>
<td>$1 + \frac{2}{3} = \frac{5}{3}$</td>
</tr>
<tr>
<td>Bob</td>
<td>BLUE</td>
<td>$1 + \frac{2}{3} = \frac{5}{3}$</td>
</tr>
<tr>
<td>Carol</td>
<td>GREEN</td>
<td>$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$</td>
</tr>
</tbody>
</table>

Smallest Piece: $\frac{1}{3}$
Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

Is there a procedure with a larger smallest piece?

Work no it
## Five Muffins, Three People–Proc by Picture

<table>
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</tr>
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<td>(\frac{5}{12} + \frac{5}{12} + \frac{5}{12} + \frac{5}{12})</td>
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**Smallest Piece:** \(\frac{5}{12}\)
Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

**Is there a procedure with a larger smallest piece?**

**NO WE CAN’T!**
5 Muffins, 3 People–Can’t Do Better Than $\frac{5}{12}$

There is a procedure for 5 muffins, 3 students where each student gets $\frac{5}{3}$ muffins, smallest piece $N$. We want $N \leq \frac{5}{12}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases.

(Henceforth: All muffins are cut into $\geq 2$ pieces.)

Case 1: Some muffin is cut into $\geq 3$ pieces. Then $N \leq \frac{1}{3} < \frac{5}{12}$.

(Henceforth: All muffins are cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: Someone gets $\geq 4$ pieces. He has some piece

$$\leq \frac{5}{3} \times \frac{1}{4} = \frac{5}{12}$$

Great to see $\frac{5}{12}$
General Problem

The Muffin Problem:
How can you divide and distribute \( m \) muffins to \( s \) students so that each student gets \( \frac{m}{s} \) AND the MIN piece is MAXIMIZED?

This Problem went from recreational Mathematics to Serious Math when we replaced \((5,3)\) with \((m,s)\).

\( f(m,s) \) be the smallest piece in the best procedure (best in that the smallest piece is maximized) to divide \( m \) muffins among \( s \) students so that everyone gets \( \frac{m}{s} \).

We have shown \( f(5,3) = \frac{5}{12} \) here.
Amazing Results!/Amazing Theorems!

1. \[ f(43, 33) = \frac{91}{264}. \]
2. \[ f(52, 11) = \frac{83}{176}. \]
3. \[ f(35, 13) = \frac{64}{143}. \]

All done by hand, no use of a computer by Co-author Erik Metz is a muffin savant!
Our First Obstacle

We solved $f(m, 3)$ and $f(m, 4)$ completely.
We solved $f(m, 5)$ **except for** $f(11, 5)$.
We had a procedure which shows $f(11, 5) \geq \frac{13}{30}$.
We have an upper bound. But they don’t match!

\[
\frac{13}{30} \leq f(11, 5) \leq \frac{11}{25} \quad \text{Diff= 0.006666...}
\]

We then showed:
$f(11, 5) = \frac{13}{30}$ using an **Exciting** new technique!
Assume that in some protocol every muffin is cut into two pieces.

Let \( x \) be a piece from muffin \( M \).

The other piece from muffin \( M \) is the \textit{buddy of} \( x \).

Note that the buddy of \( x \) is of size

\[ 1 - x. \]
There is a procedure for 11 muffins, 5 students where each student gets $\frac{11}{5}$ muffins, smallest piece $N$. We want $N \leq \frac{13}{30}$.

**Case 0:** Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both halves to whoever got the uncut muffin. Reduces to other cases.

**Case 1:** Some muffin is cut into $\geq 3$ pieces. $N \leq \frac{1}{3} < \frac{13}{30}$.

(Negation of Case 0 and Case 1: All muffins cut into 2 pieces.)
$f(11, 5) = \frac{13}{30}$, Easy Case Based on Students

**Case 2:** Some student gets $\geq 6$ pieces.

\[
N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.
\]

**Case 3:** Some student gets $\leq 3$ pieces.

One of the pieces is

\[
\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.
\]

Look at the muffin it came from to find a piece that is

\[
\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.
\]

*(Negation of Cases 2 and 3: Every student gets 4 or 5 pieces.)*
$f(11, 5) = \frac{13}{30}$, Fun Cases

**Case 4:** Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note $\leq 11$ pieces are $> \frac{1}{2}$.

- $s_4$ is number of students who get 4 pieces
- $s_5$ is number of students who get 5 pieces

\[
4s_4 + 5s_5 = 22 \\
s_4 + s_5 = 5
\]

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a **4-share**.
We call a share that goes to a person who gets 5 shares a **5-share**.
\[ f(11, 5) = \frac{13}{30}, \text{ Fun Cases} \]

**Case 4.1:** Some 4-share is \( \leq \frac{1}{2} \).

Alice gets \( w, x, y, z \) and \( w \leq \frac{1}{2} \).

Since \( w + x + y + z = \frac{11}{5} \) and \( w \leq \frac{1}{2} \)

\[
x + y + z \geq \frac{11}{5} - \frac{1}{2} = \frac{17}{10}
\]

Let \( x \) be the largest of \( x, y, z \)

\[
x \geq \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}
\]

Look at **buddy** of \( x \).

\[
B(x) \leq 1 - x = 1 - \frac{17}{30} = \frac{13}{30}
\]

GREAT! This is where \( \frac{13}{30} \) comes from!
Case 4.2: All 4-shares are $> \frac{1}{2}$. There are $4s_4 = 12$ 4-shares. There are $\geq 12$ pieces $> \frac{1}{2}$. Can’t occur.
The Future of Muffins

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1. Scott Huddleston emailed me code that ALWAYS solves the problem REALLY FAST, though he had no proof of this.
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2. Richard Chatwin independently discovered Scott’s algorithm and prove it is correct.

Richard’s paper is on arXiv.
Random Comments on Writing a Book

1. The publisher gives you a deadline. This is GOOD—I could have worked on these books forever.

2. The publisher gives you a deadline. This is BAD—I keep finding things I want to say better.

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2. The publisher gives you a deadline. This is BAD—I keep finding things I want to say better.
3. When I got a copy of my book I couldn’t stop reading it.
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3. I arranged to give a talk on Muffins at the MIT combinatorics seminar and to meet him before the talk (he lives in Boston).
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As noted earlier, Alan Frank invented the muffin problem.

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3. I arranged to speak about Muffins at the MIT combinatorics seminar and to meet him before the talk (he lives in Boston).
4. I found out from the organizers that, counting me and Alan, there should be 13 people at the talk.
5. Alan came and brought 35 muffins cut in such a way that 13 people could each get \( \frac{35}{13} \) and the smallest piece was of size \( \frac{64}{143} \).
Ask Bill Anything About the Books