Homework 01, Morally Due Tue Feb 3, 2026

1. (0 points) But DO IT anyway.
   (a) What is your name? Write it clearly.
   (b) When is the midterm scheduled (give Date and Time)? If you cannot make it in that day/time see me ASAP.
   (c) What is the Dead Cat Policy?

2. (30 point)

(a) (10 points) Let $m \in \mathsf{N}$ with $m \geq 2$.
Let COL: $\binom{\mathsf{N}}{2} \rightarrow [m]$ be

$$\mathrm{COL}(x, y) = x + y \pmod{m}$$

For $0 \leq i \leq m - 1$ let

$$A_i = \{x : x \equiv 0 \pmod{m}\}$$

Every infinite subset of an A_i is an infinite homog set.
Fill in the XXX in the following statement:
The following are equivalent

- *There is an infinite homog set that is NOT an infinite subset of one of the A_i's.*
- *m has property XXX.*

(b) (10 points) Let COL: $\binom{\mathsf{N}}{2} \rightarrow \mathsf{N}$ be

$$\mathrm{COL}(x, y) = |x - y|.$$

Find a number $X \in \mathsf{N}$ such that the following is true, and prove it:
There is a homog set of size X but not of size $X + 1$.

(c) (10 points) Let COL: $\binom{\mathsf{R} \times \mathsf{R}}{2} \rightarrow \mathsf{R}$ be

$$\mathrm{COL}(x, y) = |x - y|.$$

So COL takes two points in the plane and returns the length of the line between them.
Find a number $X \in \mathsf{N}$ such that the following is true, and prove it:
There is a homog set of size X but not of size $X + 1$.

3. (30 points) Let $x_1, x_2, \ldots$ be a sequence of reals.

Let COL: $\binom{\mathsf{N}}{2} \rightarrow [3]$ be defined by

$$\mathrm{COL}(i,j) = \begin{cases} 1 & x_i < x_j \\ 2 & x_i = x_j \\ 3 & x_i > x_j \end{cases} \quad (1)$$

(a) (30 points) Fill in the XXX in the following sentence to make it true. *Apply Ramsey's Theorem to* COL. *This gives a theorem: given any sequence of reals* $x_1, x_2, x_3, \ldots$ *there exists a subsequence (the homog set) such that XXX.*

(b) (0 points) Think about. Let $x_1, x_2, \ldots$ be a sequence of reals between 0 and 1. Using Part 1 (so using XXX) what well known theorem can you prove?

4. (20 points) Let $f\colon \mathsf{Z}\rightarrow\mathsf{Z}$. Show that there exist an infinite $\mathsf{D} \subseteq \mathsf{Z}$ such that f restricted to D (with co-domain still Z) is NOT onto.

5. (20 points) (This is not a question in Ramsey Theory but it is a prelude to one of our "Applications.")

Notation Z is the integers. $\mathsf{Z}[x_1, \ldots, x_n]$ is the set of all polynomials in $\{x_1, \ldots, x_n\}$ with coefficients in Z.

For $p(x, y) \in \mathsf{Z}[x, y]$, we view p as a function from $Z \times \mathsf{Z}$ into Z.

Note that $p(x, y)$ might be onto (e.g., $p(x, y) = x + y + 1$) or not (e.g., $p(x, y) = x^2 + y^2 + 10$).

Prove the following:

(a) FOR ALL $p(x, y) \in \mathsf{Z}[x, y]$ there is an infinite set $\mathsf{D} \subseteq \mathsf{Z}$ such that p restricted to $\mathsf{D} \times \mathsf{D}$ is NOT onto Z (that is, there is some element of Z not in the image).

(b) FOR ALL $p(x, y) \in \mathsf{Z}[x, y]$, if we let $q(x, y) = \lceil p(x, y)^{1/11} \rceil$ (Note that a number has many 11th roots, but only one real one. Take the real one.) then there is an infinite set $\mathsf{D} \subseteq \mathsf{Z}$ such that q restricted to $\mathsf{D} \times \mathsf{D}$ is NOT onto Z.

6. (EXTRA CREDIT) Prove or Disprove: FOR ALL function $p\colon \mathsf{Z} \times \mathsf{Z}$ there is an infinite set $\mathsf{D} \subseteq \mathsf{Z}$ such that p restricted to $\mathsf{D} \times \mathsf{D}$ is NOT onto Z (that is, there is some element of Z not in the image).