Homework 02, Morally Due 12:30PM, Tue Feb 10, 2026

1. (0 points) What is your name. What is your quest? What is your favorite color? What are those three questions from (do not look it up, I want to know how many know honestly).

   This HW has 5 questions for points (questions 1,2,3,4,5 though problem 1 is for 0 points) and 1 extra credit question for letters of rec and bragging rights, and to stretch your brains.

2. (25 point) We take Z to be $\{\cdots, -3 < -2 < -1 < 1 < 2 < 3 \cdots\}$.
Let $\mathrm{COL}\binom{Z}{2} \rightarrow [3]$ be defined as follows:

$$\mathrm{COL}(x, y) = \begin{cases} \mathrm{R} & \text{if } x, y \geq 1 \\ \mathrm{B} & \text{if } x, y \leq -1 \\ \mathrm{G} & \text{if } x \leq -1,\ y \geq 1 \end{cases} \tag{1}$$

Show that there is no 2-homog set $H \equiv Z$.

3. (25 point) We take Z to be $\{\cdots < -6 < -4 < -2 < 1 < 3 < 5 < \cdots\}$.
Let $\mathrm{COL}\binom{Z}{2} \rightarrow [4]$ be defined as follows:
We assume $|x| < |y|$.

$$\mathrm{COL}(x, y) = \begin{cases} 1 & \text{if } x, y \geq 1 \\ 2 & \text{if } x, y \leq -1 \\ 3 & \text{if } x \leq -1,\ y \geq 1 \\ 4 & \text{if } y \leq -1,\ x \geq 1 \end{cases} \tag{2}$$

Show that there is no 3-homog set.

4. (25 points) Find $X \in \mathsf{N}$ such that the following two statements hold.

- $\exists$ COL: $\binom{\omega+\omega+\omega}{2} \rightarrow [X]$ such that there is no X-homog $H \equiv \omega + \omega + \omega$.
- $\forall d \; \forall$ COL: $\binom{\omega+\omega+\omega}{2} \rightarrow [d] \; \exists \; (X+1)$-homog $H \equiv \omega + \omega + \omega$.

5. (25 points) Let $n \in \mathsf{N}$. The notation $n\omega$ is the ordering

$$\omega + \omega + \cdots + \omega$$

There are n copies of ω.

Find $X(n) \in \mathsf{N}$ such that the following two statements hold.

- $\exists$ COL: $\binom{n\omega}{2} \rightarrow [X(n)]$ such that there is no $X(n)$-homog $H \equiv n\omega$.
- $\forall d\ \forall$ COL: $\binom{n\omega}{2} \rightarrow [d]\ \exists\ (X(n)+1)$-homog $H \equiv n\omega$.

6. (Extra Credit) (This problem is not related to the material covered so far.) You have to do both parts.

   Let $n \in \mathsf{N}$. The number n is *jiggy* if there exists a finite collection of sets $A_1, \ldots, A_m$ such that the following hold:

   - For all $1 \le i \le m$, $A_i \subseteq \{1, \ldots, n\}$.
   - For all $1 \le i \le m$, $|A_i| = 5$.
   - For all $1 \le i < j \le m$, $|A_i \cap A_j| = 1$.
   - For all $1 \le k \le n$, there exists i, $k \in A_i$.

   This Extra Credit is in two parts. You'll see why.

   PART I: Morally due with the HW, Feb 10 at 12:30PM.

   (a) Find an infinite set of n that are jiggy.
   (b) Find an infinite set of n that are not jiggy.

   Part II: Due by the end of the semester: Characterize exactly which numbers are jiggy.