Homework 5, Morally Due Tue Mar 10, 2020, 3:30PM

1. (0 points) What is your name? Write it clearly. Staple your HW. What type of midterm will there be?

2. (35 points) The Manhattan metric is the following metric in $\mathbb{R}^2$: the distance between a point $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$ is $|p_2 - p_1|_1 = |x_2 - x_1| + |y_2 - y_1|$. (Note that this is the same as the the $L_1$ norm)

Prove the following using the 2-ary Can Ramsey Theorem:
If $X \subseteq \mathbb{R}^2$ is a countable set of points with the Manhattan metric, there exists a countable $Y \subseteq X$ such that every pair of points in $Y$ has a different distance in the Manhattan metric.

3. (35 points) (For this problem assume that there is NO cardinality between countable and the cardinality of the reals.) We say $|X| = |\mathbb{R}|$ to mean that $X$ and $\mathbb{R}$ are the same size, so there is a bijection between them.

Prove the following using a Maximal Set argument:
If $X \subseteq \mathbb{R}^3$, $|X| = |\mathbb{R}|$, no four on the same plane, there exists $Y \subseteq X$, $|Y| = |\mathbb{R}|$, such that every 4-subset of $Y$ yields a different volume.

4. (30 points) Prove that for all 2-colorings of the 5 $\times$ 5 grid, there is a monochromatic rectangle.
Recall that the 5 $\times$ 5 grid is
$\{(a, b) : a, b \in \mathbb{N} \text{ and } 1 \leq a, b \leq 5\}$
and a monochromatic rectangle is a set of four points that are the corners of a rectangle, i.e. a set of points
$\{(x, y), (x + c, y), (x, y + d), (x + c, y + d)\}$
(for some $x, y, c, d \in [5]$) that are all the same color.