1 ADD TO MISC GRAPH

DONE

The following exercise is one of Karp's original 21 problems [5].

Exercise 1.1 Let SETPACK be the following problem: Given $\{1, \ldots, n\}$, $X_1, \ldots, X_m \subseteq \{1, \ldots, n\}$, and k, are there k X_i's that are all disjoint? Show that IS $\le$ SETPACK, hence SETPACK is NP-complete.

Exercise 1.2 Let MASTERMIND be the following problem: Given a position in the game Mastermind, is there a solution? Read Stuckman & Zhang's paper [7] on the complexity of Mastermind. Rewrite their proof that VC $\le$ MASTERMIND in your own words.

2 ADD TO PLANAR CHAPTER

3 Additional Reading

DONE

We present a list of NP-hard (usually NP-complete) problems that were proven so using a known NP-complete planar problem.

- *Corral* is a paper-and-pencil puzzle involving a grid of squares where some of the squares contain natural numbers. Let CORRAL be the problem of, given an initial configuration of the Corral game, is there a way to win. Friedman [4] showed that PL-3COL $\le$ CORRAL.
- *Planar k-means (*PL K-MEANS*):* Given a finite set $S = \{p_1, p_2, \ldots, p_n\}$ of points with rational coordinates in the plane, an integer $k \ge 1$, and a bound $R \in \mathsf{Q}$ determine if there exists k centers $\{c_1, \ldots, c_k\}$ in the plane. such that

  $$\sum_{i=1}^{n} \left(\min_{1 \le j \le k} [d(p_i, c_j)]^2 \right) \le R.$$

  ($d(p, c)$ is the Euclidian distance from p to c.) Mahajan et al. [6] showed

  $$\text{PL 3SAT} \le \text{PL K-MEANS},$$

  so the problem is NP-hard, though it is not known to be in NP.

- *Multi-Robot Path Planning Problems on Planar graphs:* Given a planar graph, robots (start vertices), and destinations (end vertices), and a number k, is there a set of paths along the graph such that no two paths lead to a collision with arrival time $\leq k$? Yu [9] showed this problem is NP-hard using a reduction from Monotone PL 3SAT. For another problem from robotics that is proven hard by a reduction from PL 3SAT see Agarwal et al. [1].

- *1-in-Degree Decomposition:* Given graph $G = (V, E)$ is there a partition $V = A \cup B$ such that every $v \in V$ has exactly one neighbor in B? Dehghan et al. [2] showed (1) if G is restrictd to graphs with no cycle of length $\equiv 2 \pmod 4$ then the problem is in P, (2) if G is restricted to r-partite graphs where $r \geq 3$ then the problem is NP-complete. They use Planar 1-in-3 SAT and a monotone version of it. They have other hardness results as well.

- *The tracking paths problem:* Given $G = (V, E)$, a source $s \in V$, a destination $t \in V$, and a number $k \in \mathsf{N}$, does there exist $U \subseteq V$, $|U| \leq k$, such that the intersection of U with any $s - t$ path results in a unique sequence. Eppstein et al. [3] proved this problem is NP-complete even in the case of planar graphs using a reduction from PL 3SAT.

- In his thesis, Tippenhauer [8] discusses a myriad of variants of PL 3SAT. He covers the classic variants discussed in this book, but also looks at versions where the number of variables are bounded and the formulas are monotone.

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