

1 Related complexity problems

1. **Hardness of NCL on bounded bandwidth graphs:**

DEFINITION

Let $G = (V, E)$ be a graph and define a one-to-one correspondence $f : V \mapsto \{1, \dots, |V|\}$. The bandwidth of G is the minimum over all such mappings f of $\max_{(u,v) \in E} |f(u) - f(v)|$.

END DEFINITION

Van Der Zanden [1] showed that NCL remains PSPACE complete on graphs with bounded bandwidth.

2. **Fixed Parameter Tractability of NCL:** View the number of weight one edges, the number of weight two edges, and the number of AND/OR vertices (in a AND/OR constraint graph) as parameters to an NCL instance. Hatanaka et. al. [2] showed that NCL is fixed parameter tractable with respect to each of these parameters.
3. **Snowman is PSPACE complete:** Snowman is a SOKOBAN-like puzzle game released in 2015. He et. al. [3] showed that snowman is PSPACE complete.
4. **Reachability (Chess):** Given an $n \times n$ Chess position, is it possible to reach that position from a starting position? Brunner et al. [7] showed that this problem is PSPACE complete by a reduction from Subway Shuffle.
5. **Helpmate (Chess):** Given an $n \times n$ Chess position, is it possible to reach a position in which the black king is checkmated? Brunner et al. [7] showed that this problem is PSPACE complete by a reduction from Subway Shuffle.
6. **Motion Planning:** There are many games that fall into this category, most famously SOKOBAN. There are a handful of block pulling games in which an agent must achieve a goal by pulling (instead of pushing) blocks around. Any et. al. [4] showed that many variants of block pulling games are PSPACE complete via reductions from NCL.
7. **2048 ($n \times n$ variant) is PSPACE Hard:** 2048 is a mobile game that was popular around 2014. Rahul [8] showed this result by a reduction from planar NCL
8. **Reconfiguration of Satisfying assignments:** Given two two satisfying assignments to a planar monotone instance of Not-All-Equal 3-SAT, can one assignment be transformed into the other by a sequence of single variable flip, while ensuring that each intermediate assignment satisfies the formula? Cardinal et al. showed this problem to be PSPACE complete by a reduction from NCL.

9. **Snake-Like Robots:** Given a snake-like robot with an initial position and final position in a graph, can the robot reach the final position from the initial position without intersecting itself? Gupta et. al. [5] showed that this problem is FPT.
10. **Reconfiguration of Subset Sums:** Given two subsets S of integers with the same sum, can one subset be transformed into the other by adding or removing at most three elements of S at a time, such that the intermediate subsets always have the same sum as each other. Cardinal et al. [6] showed this problem was PSPACE complete via reductions from NCL.

References

- [1] Van Der Zanden, Tom C. “Parameterized Complexity of Graph Constraint Logic.” Application/pdf, 2015, 12 pages. <https://doi.org/10.4230/LIPICS.IPEC.2015.282>.
- [2] Hatanaka, Tatsuhiko, Felix Hommelsheim, Takehiro Ito, Yusuke Kobayashi, Moritz Muhlenthaler, and Akira Suzuki. “Fixed-Parameter Algorithms for Graph Constraint Logic,” n.d., 22.
- [3] He, Weihua, Ziwen Liu, and Chao Yang. “Snowman Is PSPACE-Complete.” Theoretical Computer Science 677 (2017): 31–40. <https://doi.org/10.1016/j.tcs.2017.03.011>.
- [4] Ani, Joshua, Sualeh Asif, Erik D. Demaine, Yevhenii Diomidov, Dylan Hendrickson, Jayson Lynch, Sarah Scheffler, and Adam Suhl. “PSPACE-Completeness of Pulling Blocks to Reach a Goal.” Journal of Information Processing 28, no. 0 (2020): 929–41. <https://doi.org/10.2197/ipsjjip.28.929>.
- [5] Gupta, Siddharth, Guy Sa’ar, and Meirav Zehavi. “The Parameterized Complexity of Motion Planning for Snake-Like Robots,” n.d., 33.
- [6] Cardinal, Jean, Erik D. Demaine, David Eppstein, Robert A. Hearn, and Andrew Winslow. “Reconfiguration of Satisfying Assignments and Subset Sums: Easy to Find, Hard to Connect.” Theoretical Computer Science 806 (February 2020): 332–43. <https://doi.org/10.1016/j.tcs.2019.05.028>.
- [7] Brunner, Josh, Erik D. Demaine, Dylan Hendrickson, and Julian Wellman. “Complexity of Retrograde and Helpmate Chess Problems: Even Cooperative Chess Is Hard.” ArXiv:2010.09271 [Cs], October 19, 2020. <http://arxiv.org/abs/2010.09271>.
- [8] Mehta, Rahul. “2048 IS (PSPACE) HARD, BUT SOMETIMES EASY,” n.d., 13.