1 Further Results

We list problems that were proven strongly NP-complete by a reduction from 3PART. You can either consider these as exercises (so to not look up the references) or recommended reading (so do look up the references). We state them as function problems; however, the reader can translate them to set problems for the reductions.

1. (Mauro Dell'Amico & Silvano Martello [6]) **Bin Packing (Natural Number Version).** Given $a_1, \ldots, a_n \in \mathsf{NP}$ and bin capacity $C \in \mathsf{NP}$, find the least number of bins B so that $a_1, \ldots, a_n$ can be packed into B bins of capacity C.

2. (Mauro Dell'Amico & Silvano Martello [6]) **0-1 Multiple Knapsack Problem.** Given (1) n items, each represented by an ordered pairs of naturals (p_i, w_i) (p_i is the profit of the ith item, w_i is the weight of the ith item), (2) m knapsacks, each represented by its capacity c_j. Pack the items into the knapsacks such that the weight in each knapsack is $\leq$ the capacity, and the profit is maximized.

3. (Jiang et al. [8]) **Minimum Common String Partition (**MCSP**)** A *partitioned string* is a string where you are also given a way to break it up. So you are given x and $(x_1, \ldots, x_m)$ where $x = x_1 \cdots x_m$. A *common partition* of strings x, y is a way to partition them so that each partition is a permutation of the other. Finally, here is the problem: Given strings x, y determine if there is a common partition, and if so then find it. They showed that the even if the alphabet size is 2, there is a reduction from 3PART.

4. (Bernstein et al. [2]) **Scheduling Problems for Parallel/Pipelined Machines.** Imagine that there are two processors P_1, P_2 and two types of jobs, so that P_j can process jobs of type j. The problem: Given n jobs $J = \{J_1, J_2, \ldots, J_n\}$ where each job is represented by $J_i = (K_i, T_i, D_i, R_i)$ where K_i is the job type, T_i is the execution time in unary, D_i is the time a job needs before it begins processing (called the Delay Time), and R_i resource requirement (this is either 0 or 1 and you cannot have two jobs with resource requirement 1 being processed at the same time). We are also given $G = (J, E)$ be the precedence graph which specifies which job should be executed before the other one. Find a way to schedule the jobs so as to minimize the the total completion time of all jobs.

5. Roemer [9] and Bachman & Janiak [1] have also looked at problems with job scheduling that are strongly NP-complete via a reduction from 3PART.

6. (Cieliebak & Eidenbenz [5]) **Measurement-errors.** Given $m, \Delta \in \mathsf{NP}$, and a multiset D of $\binom{m}{2}$ natural numbers, does there exist m points in the plane, on a line, such that the multiset of pairwise difference can be mapped to D such that if $|p - q|$ maps to d then $d - \delta \leq |p - q| \leq d + \delta$.

The above problem is additive error. They also showed that the problem for multiplicative error is reducible from 3PART.

7. (Formann and Wagner [7]) **The VLSI layout problem.** Given a graph G where every vertex has degree ≤ 4, and also given $A \in |N$, can G be embedded in a grid with area $\leq A$? (There are variants of this problem that are also strongly NP-complete.)

8. (Brimkov et al. [3]) **Matrix-similarity.** Given an upper triangular matrix A with distinct diagonal elements, $\tau \geq 1$, is there a matrix M with condition number $\leq \tau$ such that $G^{-1}AG$ is a 2×2 block diagonal matrix.

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