HW02 Problems. READDY TO GO

Exercise 0.1 We define a notion of approximte 2-coloring. Let $0 < \epsilon < 1$. The *ϵ-imperfect 2-coloring problem* is the following: The input is a graph $G = (V, E)$. Determine if there exists a 2-coloring of V such that at most an ϵ fraction of the edges have endpoints that are the same color.

Show that there exists an $0 < \epsilon < 1$ such that the ϵ-imperfect 2-coloring problem is NP-complete. (*Hint* Reduce Monotone NAE-SAT to this problem.

Exercise 0.2 Give an approximation-preserving reduction from max cut to unique coverage.

Exercise 0.3 An *apex* graph is a graph that can be made planar by the removal of a single vertex.

Given a graph $G = (V, E)$, we say that G has a *k-strong coloring* if vertices of G can be colored by at most k colors such that no two vertices sharing the same edge have the same color and every vertex in the graph dominates[1] an entire color class.

1. Show that, for all $k \geq 4$, determining if a graph G has a k-strong coloring is NP-complete.

2. An *apex* graph is a graph that can be made planar by the removal of a single vertex. Show that the problem in the first part remains NP-complete if the input is restricted to *apex* graphs.

Exercise 0.4 Given a graph $G = (V, E)$, we say the graph G is *beautiful* if we can color the vertices of G with either blue or red such that each vertex has **exactly one** blue neighbor. Either show that the exer of deciding G is beautiful is NP-hard, or show that there exists a polynomial-time algorithm for the problem.

Bonus Problem: How about when G is a planar graph?

[1] A vertex v dominates $\{v\} \cup N(v)$ where $N(v)$ is the set of neighbors of vertex v.