BILL AND NATHAN, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!!
Lower Bounds on Approx Clique Via PCP and Gaps
If $G$ is a graph then

$$\omega(G) = \text{the size of the max clique in } G.$$
CLIQUE and APPROX

We assume $P \neq NP$.

Given $G$, want to obtain $\omega(G)$.

Questions
1. Is there an alg that, given $G$, output a number $\geq \frac{1}{2} \omega(G)$?
   - NO. this is an easy exercise.
2. Is there an alg that, given $G$, output a number $\geq \frac{1}{84} \omega(G)$?
   - NO. this is an easy exercise.
3. Is there an alg that, given $G$, output a number $\geq \frac{1}{n} \omega(G)$?
   - YES. This is silly. Always output 1.
4. Is there an alg that, given $G$, output a number $\geq \log n \omega(G)$?
   - YES. This is known. This is pathetic. Can we do better?
5. Is there an alg that, given $G$, output a number $\geq \frac{1}{n^{1/2}} \omega(G)$?
   - No. We will not quite show this but will show something close.
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We will pick \( \delta \) later.
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Assume \(\text{CLIQ}\) has such an alg. We call it the approx.
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Let \(A \in NP\).

By PCP Theorem there exists \(c, d \in \mathbb{N}\) such that

\[ A \in \text{PCP}(c \lg n, d \lg n, \frac{1}{n}). \]
**Thm** \((\exists \delta < 1)\) st if there is an alg that, on input \(G\), output a number \(\geq \frac{1}{n^{\delta}} \omega(G)\) then \(P = NP\).

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By PCP Theorem there exists \(c, d \in \mathbb{N}\) such that \(A \in \text{PCP}(c \lg n, d \lg n, \frac{1}{n})\).

We use the following in a poly time program for \(A\):

1. The approx which gives \(\geq n^{-\delta} \omega(G)\).
2. The \((c \lg n, d \lg n, \frac{1}{n})\) PCP for \(A\).
Let $x \in \{0, 1\}^n$. 

We can simulate PCP on $x$ given query answers and random bits. Let $\sigma \in \{0, 1\}^{c \lg n}$. We use these as answers to queries. Let $\tau \in \{0, 1\}^{d \lg n}$. We use these as random bits. Can simulate PCP on $x$ with $\sigma \tau$. Will ACC or REJ. Simulate PCP on $x$ with $\sigma \tau$ and $\sigma' \tau'$. Either 1) ($\exists$) a query that they answer differently. Inconsistent 2) ($\forall$) queries in common they answer the same. Consistent
Preparation for Algorithm for \( A \)

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2) $(\forall)$ queries in common they answer the same. **Consistent**
Algorithm for $A$

1. Input $x$. We assume $|x|$ is a power of 2.

2. Form a graph $G$:
   1) $V = \sigma \tau \in \{0, 1\}^{c \cdot \lg n + d \cdot \lg n}$. So $|V| = n^{c + d}$.
   2) $(\sigma \tau, \sigma' \tau') \in E$ if both accept and pair is consistent.

3. $x \in A \rightarrow$ (exists) a consistent way to answer queries $\forall \tau \in \{0, 1\}^{d \cdot \lg n}$, so $\omega(G) \geq 2^{d \cdot \lg n} = n^{d}$.

4. Run the approx alg on $G$.
   1) $x \in A \rightarrow \omega(G) \geq 2^{d \cdot \lg n} = n^{d}$, so approx alg $\geq n^{d} |V| - \delta = n^{d} (n^{c + d}) - \delta = n^{d} - (c + d)\delta$.
   2) $x \not\in A \rightarrow \omega(G) \leq n^{d} - 1$, so approx alg $\leq n^{d} - 1$.

In order to make these two cases not overlap we need $d - 1 < d - (c + d)\delta$, or $\delta < \frac{1}{c + d}$.
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   3.2 $x \notin A \rightarrow$ any cons way to answer the queries will make $\leq \frac{1}{n}$ of the $\tau \in \{0, 1\}^{d \log n}$ acc. So $\omega(G) \leq n^{d-1}$.
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   4.2 \( x \notin A \rightarrow \omega(G) \leq n^{d-1}\), so approx alg \(\leq n^{d-1}\).

In order to make these two cases not overlap we need

\[ d - 1 < d - (c + d)\delta \]

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Finishing Up The Algorithm

And now back to our alg.

5. If the approx alg outputs a number $\geq n d - (c + d) \delta$ then output YES.

2. If the approx alg outputs a number $< n d - 1$ then output NO.

3. By our comments, no other case will occur.
And now back to our alg.

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   1. If the approx alg outputs a number $\geq n^d-(c+d)\delta$ then output YES.
And now back to our alg.

5. If the approx alg outputs a number $\geq n^d - (c+d)\delta$ then output \textbf{YES}.

2. If the approx alg outputs a number $< n^{d-1}$ then output \textbf{NO}.
Finishing Up The Algorithm

And now back to our alg.

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More is Known

We proved **Thm** $(\exists \delta < 1)$ st if CLIQ is $n^\delta$-approx then $P = NP$. 
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What is \(\delta\)? One could dig through the PCP machinery to find it.

\textbf{Do not bother!} The following is known.
We proved **Thm** \( (\exists \delta < 1) \) st if CLIQ is \( n^\delta \)-approx then \( P = NP \).

What is \( \delta \)? One could dig through the PCP machinery to find it.

**Do not bother!** The following is known.

**Thm** \( (\forall \delta < 1) \) if CLIQ is \( n^\delta \)-approx then \( P = NP \).
Clique is Hard to Approximate: Now What?

On this slide we assume $P \neq NP$. 

1) Yeah Very close upper and lower bounds!
2) Boo $(\log n)$ $O(1)$ $n$-approx still open.
3) Further evidence that $P \neq NP$ has great explanatory power.
4) Is this a basic problem, like $SAT$? Can we use $CLIQ$ to get other problems not approx? Alas NO, I do not know of any such results.
5) We now turn to a $SAT$-like non-approx result.
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Some thoughts on the pair of results:
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1. There exists an alg $A$ such that $A(G) \geq \frac{\log n}{n} \omega(G)$.

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3) Further evidence that $P \neq NP$ has great explanatory power.

4) Is this a basic problem, like SAT? Can we use CLIQ to get other problems not approx?

   Alas NO, I do not know of any such results.

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Some thoughts on the pair of results:

1. There exists an alg $A$ such that $A(G) \geq \frac{\log n}{n} \omega(G)$.
2. For all $\delta > 0$ there is no alg $A$ with $A(G) \geq \frac{1}{n^\delta} \omega(G)$.
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1) **Yeah** Very close upper and lower bounds!
2) **Boo** $\frac{(\log n)^{O(1)}}{n}$-approx still open.
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1) **Yeah** Very close upper and lower bounds!
2) **Boo** $\frac{(\log n)^{O(1)}}{n}$-approx still open. Nobody cares.
Clique is Hard to Approximate: Now What?

On this slide we assume \( P \neq \text{NP} \).

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3) Further evidence that \( P \neq \text{NP} \) has great explanatory power.
On this slide we assume $P \neq NP$.

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5) We now turn to a SAT-like non-approx result.