1 Edit Distance

Definition 1. Let $\Sigma$ be a finite alphabet and let $x, y \in \Sigma^*$. The edit distance between $x$ and $y$ is the number of insertions/deletions/substitutions needed to transform $x$ into $y$.

Problem 1.1. Edit Distance

INSTANCE: Two strings $x, y$ over some alphabet $\Sigma$. We think of $\Sigma$ as being fixed.

QUESTION: What is the edit distance between $x$ and $y$.

Theorem 1.

1. (Easy) Edit Distance can be computed in time $O(n^2)$ where $n = \max\{|x|, |y|\}$.

2. (Backurs & Indyk [3]) Assuming $SETH$, Edit Distance requires $\Omega(n^2)$ time.

3. (Abboud et al. [1]) With an assumption weaker than $SETH$, Edit Distance requires $\Omega(n^2)$ time.

4. (Andoni & Nosatzki [2]) For all $\epsilon > 0$ there is an algorithm that (a) runs in time $O(n^{1+\epsilon})$ and (b) returns a number that is $\leq f(\frac{1}{\epsilon})\text{OPT}(x, y)$ where $f$ is not given explicitly but is roughly double exponential in $\frac{1}{\epsilon}$.

5. (Chakraborty [5]) There exists a constant $C$ and an algorithm that (a) runs in time $\tilde{O}(n^{2-(2/7)})$ and (b) returns a number that is $\leq C\text{OPT}(x, y)$.

Is there a better quantum algorithm? Yes. Boroujeni et al. [4] proved the following.

Theorem 2.

1. (Theorem 4.5 of their paper) For all $\epsilon > 0$ there is a quantum algorithm that (a) runs in time $O(n^{2-(4/21)\log(1/\epsilon)})$ and (b) returns a number that is $\leq (3 + \epsilon)\text{OPT}(x, y)$. Note that $2 - (4/21) \sim 1.81$.

2. (Theorem 5.1 of their paper) For all $\epsilon > 0$ there is a quantum algorithm that (a) runs in time $\tilde{O}(n^{2-(5-\sqrt{17}/3)+\epsilon})$ and (b) returns a number that is $\leq O(1/\text{epsilon})^{O(1/\text{epsilon})}$. Note that $2 - (5 - \sqrt{17}/3) \sim 1.71$.

References


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