Q: Where do the edge weights come in?
A: The edge weights form a probability distribution, so the expected value is over edges. This is specified in Definition 2.2 of [HNPTW22].

Q: Is there a standard notation for the set of all qubits?
A: I am not sure if there is a field-wide standard, but it may make sense to use $V$ here, as in the set of vertices in a graph. See (**) for some comments.

Q: What are $I, X, Y, Z$ (with subscripts)?
A: $I$ is the $2 \times 2$ identity and $X, Y, Z$ are the $2 \times 2$ Pauli matrices, Pauli matrices - Wikipedia. These are operators on single qubits, so by tensoring $N$ of $\{I, X, Y, Z\}$ together we can get an operator on $N$ qubits. Something like: $X_u X_v$ is an $N$ qubit operator which acts like the identity on qubits that are not $u$ or $v$, but acts like $X$ (bit flip operator) on qubits $u$ and $v$. $H_G$ is an $N$-qubit operator composed by taking a linear combination exactly as you wrote; I don’t believe it depends on the cut, only the edge weights in $G$.

(**): Because of entanglement in the context of a quantum cut, it doesn’t really make sense to think of a function $f$ which maps each vertex to some fixed thing; what we really want to call a cut is a state on all $N$ qubits, which means you can’t talk about ”the state of qubit 23” without losing some information. It only really makes sense to take about such an $f$ in the product state case (in this case from vertices to single qubits), where we impose the restriction that there is no entanglement. The most natural way to define the general quantum max-cut problem is as: image.png (screen capture from [HNPTW22]). Questions Welcome.

Q: Reference for part 4
A: It is discussed in [HNPTW22], Theorem 2.18 (pg 12), theorem 1.2 (pg 2). Also as far as I know this is only shown under UGC and their conjecture 1.1, not $P \neq NP$ (We didn’t specify this in some places of our report - sorry!).

Q: Are there known quantum algorithms for quantum max-cut?
A: This is listed in the open questions of [HNPTW22], questions 6/8 on page 13. We briefly looked for more discussion but didn’t find any in the literature; if you want a more thorough answer we could look more into it and get back to you.

Q: Can you put this paper on arxiv?
A: We feel that some of the speculation in our paper may not be true/useful, and are hesitant to have it be one of the few things on our arxiv page. We would prefer leaving it on the class website, but if you would like then maybe we could take some time to reframe it as a review rather than an attempt at original work, and upload that?