Backdoors for SAT

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EMCL / TUD

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Backdoors in a nutshell

What?

Given a combinatorial problem, we call *backdoor variables set* (backdoors set) a set of variables that, once decided, make the rest of the problem "easy" to solve.

Why?

Backdoors have been introduced by Williams et al. ([12]) to try to explain the good performances of modern SAT solvers.

Content

Outline

Notation (1/2)

We refresh the usual notation:

- F : Formula (possibly in CNF)
- $var(F)$: the set of variables appearing in F.
- $v, \overline{v}$: a variable v or its negation $\overline{v}$
- J : (partial) interpretation. (Partial) mapping from $var(F)$ to the boolean values $\{\top, \bot\}$. We represent an interpretation compactly by listing the literals in it. Eg. $J = \{x_1, \overline{x_3}\}$
- $F|_J$: reduct of F w.r.t. the (partial) interpretation J; it is obtained by replacing each variable v in F with $J(v)$.

Notation (2/2)

Definition

Given a CNF formula F and a set of variables $V' \subseteq var(F)$ we denote with $F - V'$ the formula obtained from F by removing all the occurrences of the variables in V' from F.

Example

Given a CNF formula F and $V = \{e\}$ we obtain:

$$\begin{aligned} F - \{e\} = & a \wedge \neg b \wedge (d \vee \cancel{e}) \wedge (\cancel{\neg e} \vee \neg d) = \\ = & a \wedge \neg b \wedge d \wedge \neg d \end{aligned}$$

Outline

Class

We call *class* a set of formulas that share some property.
Well-known classes of SAT problems are:

- 2SAT:For F in CNF: $F \in 2SAT$ iff each clause of F has at most two literals
- Horn:For F in CNF: $F \in Horn$ iff each clause of F has at most one positive literal
- Renamable Horn (RHorn)
- Unit Propagation and Pure Literal (UP+PL)

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- Unit Propagation and Pure Literal (UP+PL)

Renamable Horn (RHorn)

Definition: Variable flipping

We call a *variable flipping* for the variable $x \in var(F)$, the substitution of all occurences of x in F with $\neg x$ and, similarly, of all $\neg x$ with x.

- For F CNF formula: $F \in RHorn$ iff there exists a set of variables that, once flipped, make the formula in Horn.

Example

$$F_b = (x_1 \vee x_3) \wedge (\neg x_2 \vee \neg x_1) \wedge (\neg x_3 \vee x_2)$$

$F_b \notin Horn$ but $F_b \in RHorn$ because of the flipping $\{x_1\}$:

$$(\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2)$$

Unit Propagation and Pure Literal (UP+PL)

- For F CNF formula: $F \in UP + PL$ iff it can be solved by applying only unit propagation and pure literal elimination to F

Example

$$F_c = x_1 \wedge (\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2) \wedge (x_3 \vee x_4)$$

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\begin{aligned}
F_c =& x_1 \wedge (\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2) \wedge (x_3 \vee x_4) \\
F_c|_{\{x_4\}} =& x_1 \wedge (\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2) \wedge \cancel{(x_3 \vee x_4)}
\end{aligned}
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F_c|_{\{x_4, x_1\}} =& \cancel{x_1} \wedge (\cancel{\neg x_1} \vee x_3) \wedge \cancel{(\neg x_2 \vee x_1)} \wedge (\neg x_3 \vee x_2) \\
=& x_3 \wedge (\neg x_3 \vee x_2)
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F_c|_{\{x_4, x_1\}} &= \cancel{x_1} \wedge (\cancel{\neg x_1} \vee x_3) \wedge \cancel{(\neg x_2 \vee x_1)} \wedge (\neg x_3 \vee x_2) \\
&= x_3 \wedge (\neg x_3 \vee x_2) \\
F_c|_{\{x_4, x_1, x_3\}} &= \cancel{x_3} \wedge (\cancel{\neg x_3} \vee x_2) = x_2 \\
F_c|_{\{x_4, x_1, x_3, x_2\}} &= \top
\end{aligned}
$$

Class properties (1/2)

Definition: Clause Induced

A class $\mathcal{C}$ is said to be *clause induced* whenever a formula belongs to the class iff each of its clauses (viewed as a formula) belongs to the class; i.e. $F \in \mathcal{C} \leftrightarrow \forall G_i \in F . G_i \in \mathcal{C}$

A weaker property is being closed under clause removal:

Definition: Closed under clause removal

A class $\mathcal{C}$ is *closed under clause removal* if for all formulas in the class, it holds that each subset of the clauses (when treated as a formula) belongs to the class; i.e. $\forall F \in \mathcal{C}, \forall F' \subseteq F$ it holds that $F' \in \mathcal{C}$

Class properties (2/2)

Example

Horn and 2SAT are both *closed under clause removal* and *clause induced*:

$$F_a = (\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2)$$

$F_a \in Horn$ and $\forall F' \subseteq F \in Horn$.
$F_a \in 2SAT$ and $\forall F' \subseteq F \in 2SAT$.

$G_1 = F_a$

$G_2 = (\neg x_1 \vee \neg x_3)$

$G = G_1 \wedge G_2 = (\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2) \wedge (\neg x_1 \vee \neg x_3)$

$G_2 \in Horn$, $G \in Horn$. $G_2 \in 2SAT$, $G \in 2SAT$.

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Class properties (2/2)

Example

RHorn is only *closed under clause removal* but not *clause induced*:

$$F_b = (x_1 \vee x_3) \wedge (\neg x_2 \vee \neg x_1) \wedge (\neg x_3 \vee x_2)$$

$F_b \in RHorn$ for the flipping $\{x_1\}$ and $\forall F' \subseteq F \in RHorn$ for the same flipping.

$G_1 = F_b$
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$G \notin RHorn.$

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$G \notin RHorn$.

Notational Disclaimer

Disclaimer

Some authors (eg. Szeider, Nishimura, Samer and Kottler) use "clause induced" to indicate what we call "closed under clause removal."

Subsolvers (1/2)

We are interested in classes for which there is a “good” solving algorithm:

Definition: Subsolver [12]

We call an algorithm C a subsolver if, given an input formula F:

Tricotomy: C either rejects the input F, or “determines” F correctly (as unsatisfiable or satisfiable, returning a solution if satisfiable),

Efficiency: C runs in polynomial time,

Trivial solvability: C can determine if F is trivially true (has no constraints) or trivially false (has contradictory constraint),

Self-reducibility: if C determines F, then for any assignment v of the variable x C determines $F|_{\{x \mapsto v\}}$

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Subsolvers (2/2)

There exists a subsolver for: 2SAT, Horn, RHorn and UP+PL

In the following we do not distinguish between subsolver C and related class $\mathcal{C}$.

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Outline

Strong/Weak Backdoors (1/3)

Definition: Strong C-Backdoor

A non-empty subset B of the variables of the formula F ($B \subseteq var(F)$) is a *strong backdoor* w.r.t. the subsolver C for F iff **for all** interpretations $J : B \to \{\top, \bot\}$, C returns a satisfying assignment or concludes unsatisfiability of $F|_J$.

If a formula F is satisfiable, we can define a simpler type of backdoor:

Definition: Weak C-Backdoor

A non-empty subset B of the variables of the formula F ($B \subseteq var(F)$) is a *weak backdoor* w.r.t. the subsolver C for F iff **there exists** a interpretation $J : B \to \{\top, \bot\}$ such that C returns a satisfying assignment of $F|_J$.

Strong/Weak Backdoors (2/3)

Example

Lets consider the satisfiable formula F_0:

$$F_0 = (\neg x_2 \vee x_3) \wedge (\neg x_3 \vee \neg x_4 \vee x_1) \wedge (\neg x_1 \vee x_6) \wedge (\neg x_1 \vee \neg x_6 \vee x_5) \wedge (\neg x_5 \vee x_4) \wedge (\neg x_5 \vee \neg x_6 \vee x_2)$$

$B = \{x_1, x_2\}$ is a strong 2SAT-backdoor and therefore, since F_0 is satisfiable, it is also a weak 2SAT-backdoor:

$J_0 = \{x_1, x_2\}$, $J_1 = \{x_1, \bar{x_2}\}$, $J_2 = \{\bar{x_1}, x_2\}$ and $J_3 = \{\bar{x_1}, \bar{x_2}\}$.

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$$F_0|_{J_0} = x_3 \wedge x_6 \wedge (\neg x_6 \vee x_5) \wedge (\neg x_5 \vee x_4)$$

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$$F_0|_{J_1} = x_6 \wedge (\neg x_6 \vee x_5) \wedge (\neg x_5 \vee x_4) \wedge (\neg x_5 \vee \neg x_6)$$

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$$F_0|_{J_2} = x_3 \wedge (\neg x_3 \vee \neg x_4) \wedge (\neg x_5 \vee x_4)$$

Strong/Weak Backdoors (2/3)

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$$F_0|_{J_3} = (\neg x_3 \vee \neg x_4) \wedge (\neg x_5 \vee x_4) \wedge (\neg x_5 \vee \neg x_6)$$

Strong/Weak Backdoors (3/3)

For a given formula F we define:

Definition: Minimal backdoor

A strong (resp. weak) C-backdoor B is called *minimal* iff there is no proper subset of B that is a strong (weak) $\mathcal{C}$-backdoor, i.e. $\forall B' \subset B$, B' is not a strong (weak) $\mathcal{C}$-backdoor.

Eg. The set of all variables of a SAT problem is a backdoor (for any subsolver) but, most likely, it is not minimal.

Definition: Smallest backdoor

A strong (resp. weak) C-backdoor B is called *smallest* iff it is minimal and $|B| \leq |B'|$ for any minimal C-backdoor B'

Note: There can be more than one smallest C-backdoor for the same formula F!

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Deletion (1/2)

Definition: Deletion C-backdoor [6]

A non-empty subset B of the variables of the formula F ($B \subseteq var(F)$) is a *deletion backdoor* w.r.t. a class $\mathcal{C}$ for F iff $F - B \in \mathcal{C}$.

Example

$F_0 \notin 2SAT$:

$$F_0 = (\neg x_2 \vee x_3) \wedge (\neg x_3 \vee \neg x_4 \vee x_1) \wedge (\neg x_1 \vee x_6) \wedge (\neg x_1 \vee \neg x_6 \vee x_5) \wedge (\neg x_5 \vee x_4) \wedge (\neg x_5 \vee \neg x_6 \vee x_2)$$

but $F_0 - \{x_1, x_2\} \in 2SAT$. Recall from a previous example that $B = \{x_1, x_2\}$ is a strong 2SAT-backdoor for F_0

Deletion (2/2)

The following properties make deletion backdoors interesting:

Property ([6])

If the class $\mathcal{C}$ is closed under clause removal then every deletion C-backdoor is also a strong C-backdoor (deletion $\rightarrow$ strong)

Property ([2])

If the class $\mathcal{C}$ is clause induced (eg. 2SAT or Horn) then strong C-backdoor and deletion C-backdoor are equivalent (deletion $\leftrightarrow$ strong).

Given F and $|B| = k$ we need to perform only 1 test ($F - B \in \mathcal{C}$) instead of 2^k!

Outline

Learning-sensitive (1/4)

Definition: Search tree exploration

Given a formula F, we call *search tree exploration* an ordered list of literals $(l_1, .., l_n)$ such that $l_i \in \{v_i, \bar{v}_i\}$ with $v_i \in var(F)$.

Example

Lets consider the search tree exploration $(x_1, x_2, \bar{x_3}, \bar{x_1}, x_3)$ for the following formula:

$$F = (\neg x_1 \vee x_2) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_3 \vee \neg x_1)$$

the search algorithm traversed the search space as follows:

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$$F|_{\{x_1\}} = x_2 \wedge (\neg x_2 \vee x_3) \wedge \neg x_3$$

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$$F|_{\{x_1, x_2, \bar{x_3}\}} = \perp$$

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the search algorithm traversed the search space as follows:

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the search algorithm traversed the search space as follows:

$$F|_{\{\bar{x_1}, x_3\}} = \top$$

Learning-sensitive (2/4)

How does clause learning influence backdoors?

Definition: Learning-sensitive backdoors [3]

A non-empty subset of variables B of the formula F is a *learning-sensitive* C-backdoor for F iff there exists a search tree exploration such that a clause learning SAT solver branching only on the variables in B, in this order and with C as subsolver at every leaf of the search tree, either finds a satisfying assignment or proves F unsatisfiable.

Learning-sensitive (3/4)

Example

$$
\begin{aligned}
F_1 = & (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\
& (\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\
& (\neg x \vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b)
\end{aligned}
$$

Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.

Learning-sensitive (3/4)

Example

$$\begin{aligned} F_1 = & (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\ & (\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\ & (\neg x \vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b) \end{aligned}$$

Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.

$$\begin{aligned} F_1|_{\{\bar{x}\}} = & p_1 \wedge p_2 \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge (\neg q \vee a) \wedge \\ & (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\ & (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b) \end{aligned}$$

Learning-sensitive (3/4)

Example

$$
\begin{aligned}
F_1 =&(x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\
&(\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\
&(\neg x \vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b)
\end{aligned}
$$

Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.

$$
\begin{aligned}
F_1|_{\{\bar{x}, p_1, p_2\}} = q \wedge &(\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\
&\wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b)
\end{aligned}
$$

Learning-sensitive (3/4)

Example

$$F_1 = (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge$$
$$(\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge$$
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Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.

$$F_1|_{\{\bar{x}, p_1, p_2, q, a\}} = b \wedge \neg b \wedge (\neg r \vee b) \wedge (\neg r \vee \neg b)$$

Learning-sensitive (3/4)

Example

$$F_1{}' = (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\ (\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\ (\neg x \vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b) \\ \wedge \neg q$$

Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.
Conflict: 1-UIP learning scheme gives us the clause $\neg q$

Learning-sensitive (3/4)

Example

$$\begin{aligned} F_1 = &(x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\ &(\neg q \vee a) \wedge (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\ &(\neg x \vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b) \\ &\wedge \neg q \end{aligned}$$

Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.

$$\begin{aligned} F_1|_{\{\bar{q},x\}} = &(\neg p_1 \vee \neg p_2) \wedge r \wedge (\neg r \vee a) \wedge \\ &(\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b) \end{aligned}$$

Learning-sensitive (3/4)

Example

$$
\begin{aligned}
{F_1}' = & (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\
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& \wedge \neg q
\end{aligned}
$$

Claim: $B = \{x\}$ is a learning-sensitive UP+PL-backdoor for F_1 for the search tree exploration $(\bar{x}, p_1, p_2, q, a, b, x, \bar{q}, r, a, b)$.

$$
\begin{aligned}
F_1|_{\{\bar{q},x,r,a\}} = & (\neg p_1 \vee \neg p_2) \wedge \\
& b \wedge \neg b
\end{aligned}
$$

Learning-sensitive (3/4)

Example

$$
\begin{aligned}
F_1 = & (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg p_1 \vee \neg p_2 \vee q) \wedge \\
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& (\neg x \vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b)
\end{aligned}
$$

The tree search exploration is important, since $B = \{x\}$ is not a UP+PL-backdoor if we consider x before $\bar{x}$:

$$
\begin{aligned}
F_1|_{\{x\}} = & (\neg p_1 \vee \neg p_2 \vee q) \wedge (\neg q \vee a) \wedge \\
& (\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\
& (\vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b)
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$$\begin{aligned} F_1|_{\{x, \bar{p}_1\}} = &(\neg q \vee a) \wedge \\ &(\neg q \vee \neg a \vee b) \wedge (\neg q \vee \neg a \vee \neg b) \wedge \\ &(\vee q \vee r) \wedge (\neg r \vee a) \wedge (\neg r \vee \neg a \vee b) \wedge (\neg r \vee \neg a \vee \neg b) \end{aligned}$$

at this point UP+PL is not enough to decide the formula.

Learning-sensitive (4/4)

Why are learning-sensitive backdoors interesting?

Property ([3])

There are SAT instances for which the smallest learning-sensitive UP-backdoors are smaller than the smallest strong UP-backdoor (even exponentially if the instance is unsat)

Property ([3])

There are unsatisfiable SAT instances for which one value-ordering of the variables can lead to exponentially smaller learning-sensitive UP-backdoor than a different value ordering

Trees (1/3)

Definition: Decision Tree

A binary *decision tree* is a rooted binary tree T, such that every node in T is either a leaf or has exactly 2 children. The nodes of T, except for the root, are labeled with literals s.t. the following conditions are satisfied:

- two nodes v_i and v_j with the same father are labeled with complementary literals x and $\bar{x}$;
- the labels of the node on a path from the root to a leaf do not contain the same literal twice nor a complementary pair of literals.

We call J_v the partial interpretation expressed by the path that links the root to the node v, and $var(T)$ the set of variables appearing in T.

Trees (2/3)

Definition: Backdoor Tree

A binary decision tree T (with $var(T) \subseteq var(F)$) is a *C-backdoor tree* of F if $F|_{J_v} \in \mathcal{C}$ for every leaf v of T.

Backdoor trees are interesting because of the following property:

Property ([10])

If B is a smallest strong C-backdoor for F and T is a smallest C-backdoor tree of F, then:

$$|B| + 1 \leq |T| \leq 2^{|B|}$$

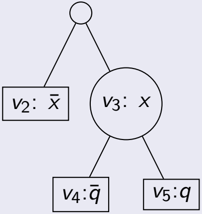
where $|T|$ is the number of leaves of T.

Trees (3/3)

Example

$$F_1 = (x \vee p_1) \wedge (x \vee p_2) \wedge (\neg x \vee q \vee r) \wedge H$$

$B_1 = \{x, q\}$ is a strong Horn-backdoor for F_1, but neither $\{x\}$ nor $\{q\}$ are. The Horn-backdoor tree for F_1 looks like:



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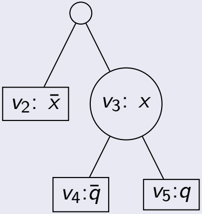

In particular, we note that there are 3 leaf nodes (v2,v4,v5) and therefore 3 partial interpretations that lead to a Horn formula: $J_{v2} = \{\bar{x}\}$, $J_{v4} = \{x, \bar{q}\}$ and $J_{v5} = \{x, q\}$.

Trees (3/3)

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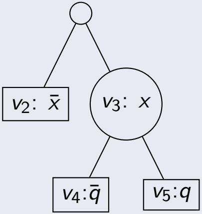

Without using the concept of backdoor trees, we would end up testing more assignments, in this case $2^{|B|} = 4$.

Trees (3/3)

Example

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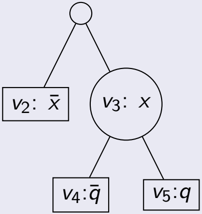

Note that variable order is important for the size of the tree, a bigger backdoor tree for F_1 is obtained by considering q before x and has 4 leaf nodes.

Outline

Complexity Intro

- We looked into classes/subsolvers that run in P,

$\Rightarrow$ Given a backdoor set for a formula, we can decide satisfiability in polynomial time

- *SAT* is NP-complete and assuming $P \neq NP \Rightarrow$ finding the backdoor set is NP-hard

Complexity Intro

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Parameterized Complexity in one Slide

- Parameterized complexity is an interesting (but not really known) field in which we "cheat" when measuring complexity
- Classical complexity: Worst-case runtime in the size n of the input
- Parameterized complexity: Worst-case runtime both in the size n of the input AND of a parameter k
- SAT: $O(2^n)$
- p-SAT: SAT parameterized by the number of variables $\Rightarrow O(2^k n^c)$, for c constant
- Algorithms that are function only of the parameter k are called *fixed parameter tractable* (FPT) : $O(f(k)n^c)$
- Observation: p-SAT is in P by considering classical complexity!
- Question: Can we find a parameter for backdoor detection to obtain a polynomial (ie. FPT) running time?

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Parameterized backdoor detection

Definition: {weak, strong, deletion, tree} C-backdoor detection

Input: A CNF formula F ;

Parameter: An integer $k \geq 0$, size of the backdoor;

Question: Does F have a {weak, strong, deletion, tree} C-backdoor of size at most k?

This problem can be solved in $O(n^k)$ for $k << n$, can we do better?

Class	Weak	Strong	Deletion	Tree
2SAT,Horn	W[2]-complete[1]	FPT[1]	FPT[1]	FPT[2]
RHorn		W[1]-hard[3]	FPT[4]	
UP,PL,UP+PL	W[P]-complete[5]	W[P]-complete[5]		

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What does this mean?

- The problems that are FPT, can be solved efficiently if the formula has a (small) backdoor of that size.
- Eg. For Horn and 2SAT we have agorithms for strong/deletion that run in $O(2^k n)$ and $O(3^k n)$ respectively([7])
- There are several techniques that have been developed for FPT problems, that might allow us to improve the above result: kernelization, iterative compression etc.

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Outline

1. Introduction
   - Notation
   - Classes and Subsolvers
2. Backdoors
   - Strong, Weak, Deletion
   - Extensions: Learning-Sensitive, Trees
3. General Results
   - Complexity Highlights
   - Experimental results
4. Conclusions

Problems in experimental data

Most of the experimental work done on backdoors does not consider the FPT complexity. Mainly two approaches are used:

- Local Search: Get a variable set, "shuffle" it until it is a backdoor, remove unneeded elements (minimize) $\Rightarrow$ cannot guarantee to find smallest backdoors, provides only an idea on the upperbound of the backdoor size.
- Complete: test every n^k subset of k variables $\Rightarrow$ easy to find smallest backdoor, unfeasible for almost any value of k

Most of the experimental work considers different classes $\mathcal{C}$: use a SAT solver as subsolver (eg. Satz) study Satz-backdoors $\Rightarrow$ hard to put results together

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Experimental overview

Nevertheless we can try to answer the question: Are backdoor sets small w.r.t. the number of variables?

- Dilkina et al. ([4]) provide a nice comparison among different classes of backdoors establishing the following "order": Horn ($9-67\%$), del RHorn ($2-66\%$),UP ($0.15-12\%$), UP+PL ($0.13-1.4\%$) and Satz ($0-0.6\%$);
- Samer and Szeider ([10]) confirm that backdoor trees are smaller than strong backdoors (for Horn and RHorn)

Experimental overview

? Random instances: Gregory et al. ([5]) show how weak UP+PL-backdoors are usually bigger in random instances (10%) than in structured SAT ($2 - 5\%$). No information is available for other classes.

? Preprocessing: The results form Dilkina et al. ([4]) are somehow inconclusive. In some cases pre-processing is "bad", in other is not relevant. Moreover they are not considering smallest backdoor but upperbounds.

Conclusions

- Backdoors allow to solve efficiently a SAT problem
- There are different types and classes of backdoors
- Finding backdoors is hard, but not-that-hard for some classes/types (FPT)
- Experimental results are hard generalize and compare. Moreover, they require an awful amount of time to be repeated.

What is next?

- Define more polynomial time classes, and study their detection complexity
- Study the concept of backdoor by removing the "efficiency" constraint
- ⋆ Apply FPT techniques to FPT detection problems to speed-up the search (eg. kernelization, parallelization and iterative compression)
- ⋆ Try to identify domains or properties of SAT instances for which C-backdoors are small, and C-backdoor detection can be solved in FPT.
- Devise heuristics that "guess" backdoors incredible well!
- Apply the backdoor idea to other domains: eg. QBF, ASP, #SAT have been covered by Szeider.

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