## The Book Review Column<sup>1</sup>

by William Gasarch Department of Computer Science University of Maryland at College Park College Park, MD, 20742 email: gasarch@cs.umd.edu

In this column we review the following books.

- 1. The Access Principle by John Willinsky. Review by Scott Aaronson. The book is about how journals should be more available on-line and other issues about journals. I invited Scott Aaronson to do a review but also told him he could make it into an opinion piece on the issue. He has done so. At the end I have a rebuttal- sort of.
- 2. A Century of Scientific Publishing: A collection of essays Edited by Fredriksson. Review by William Gasarch. This is a colletion of essays about how scientific information has been published and distributed over the last century. Can the past be helpful in understanding and dealing with our current problems?
- 3. Mathematics of Physics and Engineering by Edward K. Blum and Sergey V. Lototsky. Review by Frederic Green. This is a textbook for a 1-semester course in physics that covers much of the interplay of math and physics.
- 4. Research Problems in Discrete Geometry by Brass, Moser, Pach. Review by William Gasarch. This is a collection of research problems in discrete geometry, together with their history, context, and (lots of) references. Problems deal with points and lines in the plane, convex polygons, and many other geometrical objects.

### **Books I want Reviewed**

If you want a FREE copy of one of these books in exchange for a review, then email me at gasarches.umd.edu

Reviews need to be in LaTeX, LaTeX2e, or Plaintext.

## Books on Algorithms and Data Structures

- 1. Algorithms for Statistical Signal Processing by Proakis, Rader, Ling, Nikias, Moonen, Proudler.
- 2. Algorithms by Johnsonbaugh and Schaefer.
- 3. Nonlinear Integer Programming by Li and Sun.
- 4. Biologically Inspired Algorithms for Financial Modelling Brabazon and O'Neill.
- 5. Planning Algorithms LaValle.
- 6. Binary Quadratic Forms: An Algorithmic Approach by Buchmann and Vollmer.
- 7. Curve and Surface Reconstruction: Algorithms with Mathematical Analysis by Dey

 $<sup>^1 \</sup>odot$  William Gasarch, 2007.

#### Books on Cryptography, Coding Theory, and Security

- 1. Privacy on the Line: The Politics of Wiretapping and Encryption by Diffie and Landau (updated and expanded edition).
- 2. Concurrent Zero-Knowledge by Alon Rosen.
- 3. Cryptography and Computational Number Theory edited by Lam, Shparlinski, Wang, Xing.
- 4. Coding, Cryptography, and Combinatorics edited by Feng, Niederreiter, Xing. Formal Correctness of Security Protocls by Bella
- 5. Coding for Data and Computer Communications by David Salomon.
- 6. Block Error-Correcting Codes: A Computational Primer by Xambo-Descamps.

### **Combinatorics Books**

- 1. Combinatorics of Permutations by Bona
- 2. *Graphs and Discovery: A DIMACS Workshop* Edited by Fajilowicz, Fowler, Hansen, Janowitz, Roberts. (About using software to find conjectures in graph theory.)
- 3. Combinatorial Designs: Constructions and Analysis by Stinson.
- 4. Computationally Oriented Matroids by Bokowski

#### Auction Theory and Game Theory

- 1. Putting Auction Theory to Work by Paul Milgrom.
- 2. Game Theory: Decisions, Interactions, and Evolution by James Webb.
- 3. Superios Beings: If they exis, how would we know? by Brams. (This is a Game Theory book about Games where God may be one of the players.)

#### Logic and Verification Books

- 1. Elements of Finite Model Theory by Libkin.
- 2. Software Abstractions: Logic, Language, and Analysis by Jackson.
- 3. Formal Models of Communicating Systems: Languages, Automata, and Monadic Second-Order Logic by Benedikt Bollig.

#### **Discrete Mathematics Textbooks**

- 1. How to Prove it: A structured approache by Velleman.
- 2. Combinatorial Discrete Mathematics: Combinatorics and Graph Theory with Mathematica by Pemmaraju and Skiena.

- 3. Discrete Mathematics with Proof by Gossett.
- 4. Discrete Mathematics with Combinatorics by Anderson
- 5. Discrete Mathematics by Johnsonbaugh.
- 6. Discrete Mathematics with Graph Theory by Goodaire and Parmenter.
- 7. Discrete Mathematics without proofs, Graph Theory, or Combinatorics. This one is a joke, but the others are all real.

#### Misc Books

- 1. Automata Theory with Modern Applications by James Anderson.
- 2. Difference Equations: From Rabbits to Chaos by Cull, Flahive, and Robson.
- 3. Chases and Escapes by Nahin. (This is the math behind pursuits- if a ship is moving and another ship wants to intercept it ....)
- 4. Geometric Algebra for Computer Scientists: An Object Oriented Approach to Geometry by Dorst, Fontijne, and Mann.
- 5. *Quantum Computation and Quantum Communication: Theory and Experiments* by Mladen Pavicic.

## Review<sup>2</sup> of The Access Principle Publisher: MIT Press, 2005 Author of book: by John Willinsky Author of Review: Scott Aaronson, aaronson@csail.mit.edu

I have an ingenious idea for a company. My company will be in the business of selling computer games. But, unlike other computer game companies, mine will never have to hire a single programmer, game designer, or graphic artist. Instead I'll simply find people who know how to make games, and ask them to *donate* their games to me. Naturally, anyone generous enough to donate a game will immediately relinquish all further rights to it. From then on, I alone will be the copyright-holder, distributor, and collector of royalties. This is not to say, however, that I'll provide no "value-added." My company will be the one that packages the games in 25-cent cardboard boxes, then resells the boxes for up to \$300 apiece.

But why would developers donate their games to me? Because *they'll need my seal of approval*. I'll convince developers that, if a game isn't distributed by my company, then the game doesn't "count"—indeed, barely even exists—and all their labor on it has been in vain.

Admittedly, for the scheme to work, my seal of approval will have to *mean* something. So before putting it on a game, I'll first send the game out to a team of experts who will test it, debug it, and recommend changes. But will I pay the experts for that service? Not at all: as the final cherry atop my chutzpah sundae, I'll tell the experts that it's their professional duty to evaluate, test, and debug my games for free!

<sup>&</sup>lt;sup>2</sup>©Scott Aaronson 2007

On reflection, perhaps no game developer would be gullible enough to fall for my scheme. I need a community that has a higher tolerance for the ridiculous—a community that, even after my operation is unmasked, will study it and hold meetings, but not "rush to judgment" by dissociating itself from me. But who on Earth could possibly be so paralyzed by indecision, so averse to change, so immune to common sense?

I've got it: academics!

Everything I described with computer games would work even better with academic papers. For then it wouldn't be the academics themselves who were footing the bill, but their universities' libraries. So, under the academics' noses, I could gradually gain control of much of the world's scientific output—a unique and irreplaceable resource, worth almost any price I'd care to name.

Alas, my idea has already been taken, by Elsevier and the other publishing conglomerates. At the risk of stating the obvious, we in the academic community create the ideas in our papers. We write the papers. We typeset the papers. We review the papers. We proofread the papers.<sup>3</sup> We accept or reject the papers. We electronically archive and distribute the papers. If commercial publishers once played an essential role in this process, today their role is mostly to own the copyrights and to collect money from the universities.

In the past few years, there have been many detailed analyses of the rise in journal prices over time, the cost per page of one journal versus another, the tactics of the publishing companies, and so on. A website by John Baez [1] and an open letter by Donald Knuth [3] provide excellent starting points for those who are interested. In my view, what's missing at this point is mostly *anger*—a justified response to being asked to donate our time, not to Amnesty International or the Sierra Club, but to the likes of Kluwer and Elsevier. One would think such a request would anger *everyone*: conservatives and libertarians because of the unpaid labor, liberals because of the beneficiaries of that labor.

But scientists, despite (or because of) their professional virtues—understatement, self-criticism, respect for academic tradition—seem prone to a peculiar anger deficiency. Not only do many of them continue to work *pro bono* for outrageously-priced journals, some of them even criticize colleagues who don't! Lance Fortnow recently defended Elsevier's *Information and Computation*, a subscription to which costs a jaw-dropping \$3000 per year, as follows:

We all have a responsibility to do our fair share of refereeing and it takes no more effort to referee a paper for I&C than for any other journal. If you truly dislike a certain publisher then don't submit your papers to their journals. But to take a symbolic stand by not refereeing papers only hurts the authors and our community. [2]

In my view, once we've mustered a level of anger commensurate with what's happening, we can *then* debate what to do take next, which journals are overpriced and which aren't, what qualifies as "open access," and so on. But the first step is for a critical mass of us to acknowledge that *we are being had*.

This article is supposed to be a review of a book called *The Access Principle* by John Willinsky (MIT Press, 2006). So let me now turn to reviewing it. *The Access Principle* is a paradox: on

 $<sup>^{3}</sup>$ Most journals do offer a proof reading service; some of these have achieved legendary status for introducing more errors than they fix.

the one hand, its stated goal is to make the case for open access to research and scholarship. Its thesis is that "a commitment to the value and quality of research carries with it a responsibility to extend the circulation of such work as far as possible and ideally to all who are interested in it and all who might profit by it" (p. xii). On the other hand, the book is printed in hardcover and sells for \$34.95. Recognizing what he calls the "all-too-obvious irony," Willinsky explains that while much of the book's content is available for free online, he's chosen to collect it in book form, first, to reach a wider audience; second, because of his "admitted attachment to the book's becoming look and familiar feel"; and third, because "the book remains the medium that best serves the development of a wide-ranging and thoroughgoing treatment of an issue in a single sustained piece of writing" (p. xiv-xv). Fair enough—in any case, my review copy was free.

For me, the most important idea in the *The Access Principle* is that scholars have a duty to make their work available, not only to their colleagues, but ideally to *anyone who wants it*. As Willinsky writes:

Open access holds the promise of moving knowledge from the closed cloisters of privileged, well-endowed university campuses to institutions worldwide. Such an approach also opens a new world of learning to those outside the academic realm, to dedicated professionals and interested amateurs, to concerned journalists and policymakers. In this way, an open access approach to scholarly publishing is not simply a side issue, a matter of business plans and delivery systems, in the pursuit of truth. (p. 33)

Today, many journal articles are online, but are accessible only from schools, companies, and research centers that have bought exorbitantly-priced "institutional subscriptions" to services like Elsevier's ScienceDirect. I've always been amazed by the arrogance of the view that this represents an acceptable solution to the problem of circulating research. Even if the subscriptions cost a reasonable amount (they don't), and even if the researchers who were "entitled" to them could easily access them away from their workplaces (they can't), who are we to say that a precocious high-school student, or a struggling researcher in Belarus or Ghana, has no legitimate use for our work? Or if our work is intended only for a small circle of colleagues, then why even bother writing it up? Why invest months of boring, painstaking effort to express, in elegant LaTeX form, what would probably take fifteen minutes to explain to a colleague on a blackboard? How serious are we about scholarship being an eternal conversation that transcends time and space?

The first time I saw a college library, I was eleven years old and attending a summer program at Bucks County Community College. If you remember the scene from Disney's "Beauty and the Beast" where the provincial Belle sees the endless shelves of books in the Beast's library, you'll know roughly how I felt. Even though the Bucks library was tiny by research standards, nothing had prepared me for it—certainly not my school library or the local public one. I never knew that so many words had been written about such esoteric topics. When I picked up a recreational math journal, and found an article about generalizing the Fibonacci sequence to "Tribonacci" and higher-order sequences, I felt like I was entering a secret world.

Granted, it might not be feasible for every elementary school on Earth to stock journals containing articles about the Tribonacci sequence. The point is that today, in the Internet age, they shouldn't have to. And yet, even as I write, much of the serious content on the Internet remains sequestered behind pointless, artificial walls—walls that serve the interests of neither the readers nor the authors, but only of the wall-builders themselves. If I have a medical problem, why can't I download the full text of clinical studies dealing with that problem? Why do so many researchers still not post their papers on their web pages—or if they do, then omit their early papers? When will we in academia get our act together enough to make the world's scholarly output readable, for free, by anyone with a web browser?

Since this is a book review, at this point I have to level with you. Apart from an excellent final chapter—which describes the founding of the *Transactions of the Royal Society of London*, the world's first scientific journal—*The Access Principle* is an almost unreadably boring book. Rather than try to explain *why* it's boring, I'll simply ask you to read the following two sentences.

The current state of serial indexing presents a particularly good reason for research libraries and professional associations, as well as individual researchers and journal editors, to work together on developing compatible distributed systems that greatly improve the comprehensiveness of indexing and promote universal access to research by placing at least this initial, discovery phase of scholarship squarely within the public sector of the knowledge economy. Comprehensive indexing may be an area in which commercial and open access interests can coexist peacefully, complement one another, and even thrive and serve one another, as the future of scholarly publishing sorts itself out within this new digital medium. (p. 187)

Now imagine 243 pages of prose like the above, and you'll understand why *The Access Principle* isn't going to fly off the shelves, despite the timeliness and importance of its message. And yet, even if he seems physically unable to write one subordinate clause where five would do, I'm grateful to Willinsky all the same—for in *The Access Principle*, he's given the open-access movement its first attempt at an intellectual foundation. Now it's up to the rest of us to supply the anger.

# References

- J. Baez. What We Can Do About Science Journals, 2006. http://math.ucr.edu/home/baez/journals.html.
- [2] L. Fortnow. A Referee's Boycott, weblog entry, February 16, 2006. http://weblog.fortnow.com/2006/02/referees-boycott.html.
- [3] D. Knuth. Open Letter to the Journal of Algorithms Editorial Board, October 25, 2003. http://www-cs-faculty.stanford.edu/~knuth/joalet.pdf.

## Some Counter Arguments to Aaronson's Arguments Elsevier Publishers

### Some Comments by Bill Gasarch

You may have noticed that the last section is blank. This is not a typo. I invited someone from Elsevier to write a rebuttal, or to write their own review. They said they would but never got around to it. There is a website of intelligent commentary on this issue to be found here: http://www.earlham.edu/peters/fos/

# A Review<sup>4</sup> of A Century of Scientific Publishing: A collection of essays Edited by Fredriksson IOS press Review by William Gasarch gasarch@cs.umd.edu

# 1 Introduction

There is a debate going on right now about the future of academic publishing. Some of the questions being raised are as follows.

- 1. Since authors and referees do all the work for no pay, why are journals so expensive?
- 2. Should all journals be on-line? If they are, then will people still subscribe to them.
- 3. Do we need journals in their current form?

I thought that a book on the history of Scientific Publishing would give some insights, at least in terms of telling where we've been. A historian once cautioned against taking analogs too seriously, but said instead that history is important to study because "when you go to a foreign country and then come back to your native land, you see things you would not have seen before." Hence I read this book. Did it help? Not really, though it did have somethings of interest.

The articles fell into two categories.

# 2 Straight History

The first 14 chapters are on the history of publishing. Some are on particular publishers, some are on Countries, and some are on people. There were a few interesting tidbits here and there-Descartes played a big role in getting scientific publishing started, Germany had a big decline in it because of WW II, at various times referees were paid to have articles reviewed in a timely manner, but for the most part these chapters were not that interesting. The biggest problem (for me) is that this was history without any interpretation. A typical article was reminiscent of the famous quote 'History is just one damn thing after another'

# 3 Tools and Trends

There were 11 chapters on Tools and Trends. These included discussions of electronic tools, TeX, Peer Review, Refereeing in general, e-journals. While some of what it said was interesting, it seems to already be out of date. For example, none of the issues raised above were discussed.

<sup>&</sup>lt;sup>4</sup>©2006, William Gasarch

# 4 Opinion

This book has done a good job of gathering facts and issues, but not of interpreting them. A good book could be written that also tells us what to make of all this. The relevance of History to our current problems may be limited.

Review<sup>5</sup> of Mathematics of Physics and Engineering Authors: Edward K. Blum and Sergey V. Lototsky Published by World Scientific \$75, Hardcover, 482 pages

Review by Frederic Green (fgreen@black.clarku.edu)

# 1 Overview

This text is designed for a course typically given in the junior or senior year, addressed to scientists (especially physicists and chemists) and engineers, detailing various topics in mathematics that arise in those disciplines. Generally these courses assume some knowledge of physics, and a mathematical background that consists of an introductory sequence through multivariate calculus, and first courses in linear algebra and differential equations. Major topics typically include vector analysis, complex analysis, Fourier analysis, and partial differential equations. The popular and/or classic texts that come to mind in this category include, for example, Arfken and Weber's Mathematical Methods for Physicists, Boas's Mathematical Methods in the Physical Sciences, or Kreyszig's Advanced Engineering Mathematics. Although it plays a similar role, Blum and Lototsky takes an unconventional approach. The topics are more focussed and many are out of the scope of these other textbooks. In a nutshell, it puts much greater emphasis on the interplay between mathematics and physics, and goes to great lengths to involve the reader. This is discussed at some length in the last section of this review.

# 2 Summary of Contents

## Chapter 1: Euclidean Geometry and Vectors

The foci of this chapter are elementary notions of geometry, vectors, and kinematics. The chapter begins with a quick review of Euclidean geometry. The idea of a reference frame is also introduced, and is returned to again and again over the next 3 chapters. Using this solid conceptual basis, vectors and associated operations such as the inner and cross products are defined and discussed. Curves in  $\mathbb{R}^3$  are introduced as vector functions of a scalar variable, and the description of the curve's points and corresponding tangents is given via the Frenet formulas. The chapter ends with the definition of velocity and acceleration, giving basic properties and expressing them in different coordinate systems.

## Chapter 2: Vector Analysis and Classical and Relativistic Mechanics

The subject of dynamics is taken up in earnest. Newton's Laws of motion are formulated and

 $<sup>^{5}</sup>$  ©Fred Green, 2007

re-expressed in different reference frames, both inertial and non-inertail. A consequence of the latter, in the context of uniformly rotating frames, are clear expositions of interesting phenomena such as Foucault's pendulum and the Coriolis force. Transformations between generally accelerated frames also provide a foundation for the later treatment (in the same chapter) of general relativity. Systems of point masses, both rigid and non-rigid are discussed, along with the requisite concepts of angular momentum and moments of inertia. The Euler-Lagrange and Hamiltonian formalisms of classical physics receive a succinct treatment. The development then moves on to relativistic mechanics. Enough of the theory of special relativity is explained to obtain results like  $E = mc^2$  and the Lorentz-Fitzgerald contraction. The Einstein field equations for general relativity are then postulated; this is one (very understandable) instance in which the physical origin of the equations is not explained in detail. While it is nice for the reader to see the equations even without extensive explanation, and the explanations given are by in large correct, there are some inaccuracies, which are explained in the "Evaluation" section. The section is nevertheless a good quick (!) introduction to general relativity, going so far as to give the Schwarzschild solution and some of its most important consequences, including part of the theory of black holes.

### Chapter 3: Vector Analysis and Classical Electromagnetic Theory

The mathematical thread running through this chapter is, not surprisingly given its title, classical vector field theory. It begins with basic definitions about vector functions, scalar and vector fields, and the gradient. It introduces line and surface integrals, and the divergence and curl operators. An appealing feature is the use of elementary notions of measurability and limits (still within a very concrete framework) to write *coordinate-free* definitions of div and curl. It is also explained how these operators can be expressed in arbitrary orthogonal curvilinear coordinate systems. Next the central integral theorems of vector analysis are presented: the theorems of Green (no relation!), Stokes, and Gauss. The physical intuition underlying these theorems is explained and, as with most of the major theorems in the text, the proofs are given as exercises. The mention of the generalized version of Stokes's Theorem from differential geometry, for which much of the intuition is established in this (and the previous) chapter, is a nice touch. One application of Gauss's theorem is given via an introduction to potential theory (in particular, properties of Laplace's and Poisson's equations). The chapter culminates in a section on Maxwell's equations. They are derived from physical principles (e.g., Coulomb's Law and Ampère's Law) as well as the earlier mathematical results in the chapter (e.g., Gauss's Theorem). Some static solutions are presented, including those for fields surrounding electric and magnetic dipoles; dynamic solutions are put off until Chapter 6. Maxwell's equations in material media are also given (up to notation, the same as those that hold in the vacuum); the main emphasis here is physical, but it seems the authors are not letting any opportunity pass to point out the uniformity of the mathematical framework. The observation that the vector potential is not unique, thus hinting at gauge invariance (which is discussed in a little more detail in chapter 6) is a welcome feature.

#### **Chapter 4: Elements of Complex Analysis**

This chapter largely leaves physics and engineering behind for a time, and gives a fairly standard basic treatment of functions of a complex variable. The definitions of complex variables and the complex plane, along with some brief history, followed by an example using complex numbers to analyze AC circuits<sup>6</sup>, all provide good motivation for exploring the theory. Functions of a complex variable and analytic functions are defined. An example is given to motivate the Cauchy-Riemann equations, and again the reader is invited to participate in the proof that they are an equivalent criterion for differentiability. This is followed by Cauchy's Integral Theorem and Integral Formula. Cauchy's Integral Theorem is used, among other things, to prove the Fundamental Theorem of Algebra. Conformal mappings are explained and used as a tool for the analysis of Laplace's equation in two dimensions. We proceed to power series, convergence, and Taylor series as a characterization of analyticity. Next come the Laurent series, various types of singularities including poles, the Residue Theorem and some fine examples of residue integration. There is brief mention of branch points, but branch cuts and Riemann surfaces are not discussed. There is a section on power series solutions of ordinary differential equations, the emphasis being on complex solutions near singular points.

## **Chapter 5: Elements of Fourier Analysis**

The chapter first lays foundations, then proceeds (more or less) to successively more general methods. Basic definitions and properties of Fourier series and coefficients are given, along with a brief exposition of Bessel's Inequality and Parseval's Identity. Point-wise convergence is contrasted with uniform convergence, for which the Weierstrass M-test is given, as is a sufficient condition for a function to have a Fourier series. Motivational and historical notes here provide a good orientation for the reader; this includes the discussion of the connection between Fourier series and signal processing, and a physical interpretation of Parseval's Identity. After some applications to ordinary differential equations, the text moves on to the (continuous) Fourier transform. This is investigated in much the same spirit and using similar methodologies to that used for the Fourier series. An introduction to the discrete Fourier transform includes well-motivated brief introductions to the Dirac  $\delta$ -function and the fast Fourier transform. The final section of the chapter investigates the Laplace transform, including a section giving applications to system theory, a good illustration of the chapter's techniques.

## Chapter 6: Partial Differential Equations of Mathematical Physics

This is by far the longest chapter, and taking up almost a quarter of the book it covers quite a bit of ground in the field of classical solutions of partial differential equations (PDEs). The first section uses simple examples to illustrate the more general solution techniques that follow: variation of parameters is used to solve the transport equation, and Fourier analysis and separation of variables for the heat and wave equations. Some of the physical derivations of these equations are included. There follows an introduction to the general theory of PDEs. This includes some methods of classification, the method of characteristics, variation of parameters, and separation of variables. The techniques are given adequate detail, are nicely summarized and illuminating examples are given, although it is a little surprising that the phrase "variation of parameters" is left unexplained. A long section considers various classical PDEs. These include the telegraph, Helmholtz, wave and Maxwell's equations, as well as some equations of fluid mechanics including Navier-Stokes. For Maxwell's equations, there is some discussion of gauge invariance and gauge fixing, and the propagation of electromagnetic waves is derived. The next section turns to equations of quantum mechanics, focussing on Schrödinger's equation. As in the case of general relativity,

<sup>&</sup>lt;sup>6</sup>Notation alert for a sizable fraction of SIGACT readers: here "AC" really stands for "alternating current"!

adequate justification can hardly be expected, but there is a very good sketch of the history and results leading up to the equation. The postulates of quantum mechanics are stated and some solutions of Schrödinger's equation are given, one quite simple (the harmonic oscillator) and one not so simple (the hydrogen atom). In the latter case, building on the techniques developed earlier, all energy levels are computed in the non-relativistic approximation. The other "equation of quantum mechanics" that is considered is the Dirac equation. This includes Dirac's magical derivation of the equation, some explanation of the nature of intrinsic spin, and a brief hint as to the existence of anti-matter. Included in the section on quantum mechanics is a sub-section on quantum computing. The material includes, early on, the Deutsch algorithm (for evaluating  $f(0) \oplus f(1)$  with only one query to f, stated here as an exercise), with a later hint at its generalization, the Deutsch-Josza algorithm. The basic ideas (qubits, universal sets of quantum gates, entanglement, quantum algorithms) are defined and discussed. There are brief discussions of Grover's and Shor's algorithms, but no technical details. The final section of Chapter 6 is a survey of numerical methods for PDEs. The numerical quadrature problem is defined, and explicit and implicit methods and stability are illustrated via ordinary differential equations. Finite difference methods are applied to the heat, wave and Poisson equations. The chapter concludes with an introduction to the finite element method.

#### **Chapter 7: Further Developments**

This is a set of problems, some of them quite substantial, that extend material in the text proper. For example, one problem leads the reader through the calculation, drawing on the Schwarzschild solution of Chapter 2, of the precession of the perihelion of Mercury. Another is an analysis of the Michelson-Morley experiment. There is a problem that entails an introduction to quaternions, and another that works through the proof of point-wise convergence of Fourier series. There is a long exercise on the 1D Sturm-Liouville problem, and a shorter one on solitons in the Korteweg-deVries equation (this is a nonlinear PDE that describes shallow waves, and the soliton phenomenon has many important analogs in other areas of physics). There are many others of a similar nature.

**Chapter 8** is an appendix that includes, most notably, review material on linear algebra, ordinary differential equations, and tensors.

# **3** Evaluation and Opinion

Considering the sheer bulk of other volumes in this category (e.g., Kreyszig's text weighs in at 1248 pages), it is refreshing to encounter a book such as Blum and Lotosky, which isn't any harder to lift than the average novel. Part of the reason is that it makes no attempt to be encyclopedic, being designed for a one-semester course. But far more importantly, the book largely motivates and outlines the subject, leaving it up to the reader to provide much of the substance. Exercises are tightly integrated with the text, with the proofs of major theorems (in part or in their entirety), and significant calculations, being stated as exercises for the reader, generally given with ample hints. Those who work through all the exercises are bound to feel more like a *participant* than a passive reader, and indeed might almost end with the impression that they contributed to the writing of the book. If this isn't sound mathematical pedagogy, I don't know what is.

It is also distinguished by the tight integration of mathematics and its applications (largely physics). Many physical principles are introduced virtually from scratch to derive the relevant

mathematics, and the mathematics in turn is used to derive physical results, so often it reads more like physics than mathematics. Very important physical ideas (e.g., relativity and quantum mechanics) are included that are usually omitted in conventional books in this category.

There are, on the other hand, a number of inaccuracies, shortcomings or quibbles I feel compelled to point out:

- It is stated on page 106 that the Einstein equations describe "the relation between the metric tensor and the gravitational field..." Actually, the metric *replaces* the gravitational field; the Einstein equations give the relation between the metric and the *matter* fields as embodied in the energy-momentum tensor. (In a future edition the authors may consider including Wheeler's incomparable aphorism, "matter tells space-time how to curve, and curved space tells matter how to move;" this description is indeed enumerated in somewhat greater detail at the bottom of page 110.) Later, on page 118, in the course of deriving the (correct!) gravitational red-shift, it is erroneously stated that a certain photon of frequency  $\nu$  has mass  $h\nu/c^2$ . All photons have zero mass. What is true is that a massive particle can *lose* a mass of  $h\nu/c^2$  by emitting such a photon. This fact can be used, together with the principle of equivalence, to give a correct argument (as is done in some books on general relativity <sup>7</sup>).
- In Chapter 3, there is a misrepresentation that ought to be addressed, even though it is only mentioned in passing. On page 178 (and again on page 351) the authors describe Yang-Mills theory as a "quantum-theoretic analog of Maxwell's equations." In fact, Yang-Mills theory can be formulated classically. Like Maxwell's theory, the classical Yang-Mills theory must be quantized to obtain an analog of quantum electrodynamics. It is far more accurate to say that the Yang-Mills equations are analogs of Maxwell's equations because the gauge group, U(1) in the case of Maxwell, becomes non-abelian (and, with a little more effort, this could be stated in a more accessible manner to the intended readers of the book).
- In Chapter 4, I felt a good opportunity was missed in not including the application of conformal mappings to simple problems of fluid flow and/or electrostatics, which would have fit very nicely with the overall aims of the text.
- In Chapter 5, the section on system theory does not contain any explanation of the terminology or motivation – quite at odds with the rest of the book.
- The section on quantum computing in Chapter 6 is a reasonable sketch, but it is oddly placed, since (insofar as it is covered here) it has no direct relationship with PDEs.
- Generally, there could be many more figures, and the language is occasionally a bit stilted.

Despite these problems, none of which are major, this is a good text. It is very readable, using an engaging narrative style and a healthy sprinkling of biographical and historical background throughout. As the authors promise, the exercises do indeed have an "element of fun," and it is a rewarding book to work through.

<sup>&</sup>lt;sup>7</sup>E.g., in his book *Gravitation and Cosmology* (pg. 85), Weinberg remarks that the calculation works if one ascribes a "gravitational potential energy" to a photon, but also notes that without an emitting particle such a concept is "without foundation."

A final word: This book is, to say the least, not typical of those generally reviewed on these pages. No doubt many readers of SIGACT news will ask "why should I (or any of my students) read it"? Here are some possible answers. One is that some of you may be employed in math departments where you will on occasion have to teach a course such as this. In that case, consider this text for adoption. Others, whose background is more purely in the computer science or mathematics camp, might simply be curious to learn more about mathematical physics. Consider that this book is primarily written from a mathematical point of view. Therefore, if your physics education is non-existent or has serious holes in it, you may very well want to start here.

A Review<sup>8</sup> of Research Problems in Discrete Geometry Authors of Book: by Brass, Moser, Pach Springer-Verlag Review by William Gasarch gasarch@cs.umd.edu

# 1 Introduction

This is a book that gathers together an enormous number of open problems in Discrete Geometry. The problems are organized into categories and presented with historical context, results, many references, but few proofs. This is just as well– the book is already 499 pages. The point of this book is not to read it and learn how to prove a theorem. The point of the book is become interested in a problem and perhaps read some references. The intended audience is both researchers and graduate students; however, I will comment more on the readership in the last section of the review.

# 2 Summary of Contents

This is an odd book to give a "summary of contents" to since the book itself is a "summary of a field". Hence I will (1) be informal, (2) stick to  $R^2$ , (3) not attempt to be complete, (4) not use the books division of types of problems, and (5) try to give problems that do not require too many definitions. I will also give my thoughts on these problems in italics.

There is a website of of up-to-date information about the problems in this book:

http://math.nyu.edu/pach/research\_problems.html

(There is a tilde before 'pach' and an underscore between 'research' and 'problem'.)

# 2.1 Packing, Covering, and Tiling Problems

The first four chapters are on packing, covering, and tiling. These problems all involve arrangements of certain geometric objects in the plane. A *packing* is an arrangement where the objects are not allowed to overlap; however, you might not cover the entire plane. The goal is to maximize how much you cover. A *covering* is an arrangement where every point must belong to at least one object; however, the objects may overlap. The goal is to minimize the amount of overlap. A *tiling* is an arrangement which is both a packing and a covering. The goal is to just get it done.

<sup>&</sup>lt;sup>8</sup>©2007, William Gasarch

**Definition:** Let C be is a set of shapes in the plane (e.g., a set of discs of diameter 1) and D is a bounded area (e.g., a  $10 \times 10$  square).

- 1.  $\delta(C, D) = \frac{\sum_{x \in C} AREA(x \cap D)}{AREA(D)}$ .
- 2. If  $D = R^2$  then  $\delta(C, D) = \lim r \to \infty \delta(C, B(r))$  where B(r) is the disc of radius r centered at the origin. (If this limit does not exist one can look at limsup and liminf.)

We first state a known result to give a feel for the area. What is the maximum density that can be achieved when packing the plane with nonoverlapping disks. Thue thought he had a proof that it was  $\delta \leq \frac{\pi}{12}$ . His original proof was not correct; however, he later found a proof that is correct. Other proofs are known, some of them moderately easy.

1. What is the maximum density that can be achieved with packing the plane with nonoverlapping semidisks? There is a packing with

$$\delta(C, R^2) = \frac{\pi}{\sqrt{3} + 5 \tan \frac{\pi}{10}} > \frac{\pi}{\sqrt{12}}.$$

This is better than Thue's result for disks. Hence it is not optimal to make the semidisks into disks and use them. This would seem to be something people would know, but wow, its still open and their have been lots of papers on it!

- 2. What convex shapes are *bad* at packing the plane? That is, what convex shape C is such that  $\delta(C, R^2)$  is as small as possible. It is known that, for every convex C,  $\delta(C, R^2) \ge \sqrt{2}/2$ .
- 3. There are also open problems having to do with restricting C to be a variety of shapes, looking at certain types of coverings, and also looking at how many of that convex shape you need.
- 4. Can one pack all squares of sides  $\frac{1}{i}$  into a rectangle of area  $\frac{\pi^2}{6} 1$ ? Note that this is the exact area you need, so it would actually be a tiling.
- 5. Determine the constant A such the following holds: For any (possibly infinite) set of squares of total area 1, there is a rectangle of area A into which they can be packed. It is known that  $\frac{2+\sqrt{3}}{3} \leq A \leq 1.53$ . This is a great question!
- 6. Determine which convex polygons C are such that the plane can be tiled with congruent copies of C. Is this problem (if phrased properly) decidable? If it is decidable then how efficiently can we do it? Most of the book is essentially pure math, but this shows how computer science has influenced math. In another era they would have asked for a 'characterization of all such C', instead of an algorithm.

### 2.2 Points and Lines in the Plane

1. Given n points in the plane what is the maximum number of equal distances that you can have? Call this number u(n). The values  $u(1), \ldots, u(14)$  are known. Asymptotically it is known that

$$\Omega(ne^{\frac{c\log n}{\log\log n}}) \le u(n) \le O(n^{4/3}).$$

Nothing more is known, so the open problem is to improve either the upper or lower bound. Erdős conjectured that  $u(n) \leq O(n^{1+\epsilon})$ . This problem has also been looked at when you assume the points are in general position (no three colinear), and other restrictions (e.g., no four on a circle).

I'm surprised I had not heard of this problem. Looks nice!

2. Given n points in the plane what is the maximum number of distinct distances that you can have? Call this number v(n). The values  $v(1), \ldots, u(13)$  are known. Asymptotically it is known that

$$\Omega(n^{0.864...}) \le v(n) \le O(\frac{n}{\sqrt{\log n}}).$$

Nothing more is known, so the open problem is to improve either the upper or lower bound. Erdős conjectured that  $u(n) = \Theta(\frac{n}{\sqrt{\log n}})$ . This problem has also been looked at when you assume the points are in general position (no three colinear), and other restrictions (e.g., no four on a circle).

3. The Szemeredi-Trotter theorem states the following: For any set of n points and m lines in the plane, if I is the number of incidences then  $I = O(n + m + (nm)^{2/3})$ . The original proof was difficult; however, there are now easier proofs. The constants have been pinned down to a remarkable degree. The following is known:

$$(0.42(nm)^{2/3} + m + n \le I \le 2.5(nm)^{2/3} + m + n)$$

Can the gap between the coefficients of the  $(nm)^{2/3}$  be improved. This theorem has many applications so this may be important. Then again, it may not. I'm really surprised the gap is so small!

4. It is known that, given 5 points in the plane in general position (no three collinear) there exists 4 of them that form a convex 4-gon. More generally, let f(r) be the least number such that if there are f(r) points in the plane in general position then some r of them will form a convex r-gon. It is known that

$$2^{r-2} + 1 \le f(r) \le \binom{2r-5}{r-2} + 1.$$

The open problem is to narrow or close the gap. The conjecture is that the lower bound is the right value. f(3) = 3, f(4) = 5, and f(5) = 9 but nothing else. This is the Erdős-Szekeres Problem. This problem I've heard of and read a good survey of, so this did not tell me anything new. But thats just me.

5. Given n points in the plane, not collinear, there will always be ol(n) pairs of points that have the line going through them not hit any other of the n points. What is ol(n)? It is known that, if n is even,

$$\frac{6n}{13} \le ol(n) \le \frac{n}{2}$$

(if n is odd then the upper bound is  $3\lfloor n/4 \rfloor$ ). The open problem is to narrow or close the gap. The conjecture is  $\frac{n}{2}$ . This is the Sylvester-Gallai Problem.

## 2.3 Coloring Problems

- 1. The following are known:
  - For all 3-colorings of  $\mathbb{R}^2$  there exists two points that are an inch apart and have the same color.
  - There exists a 7-coloring of  $\mathbb{R}^2$  such that no two points that are an inch apart are the same color.

It is a long-standing open problem to close this gap. Someone once told me this is the most important open problem in mathematics. His point was more that this is a problem you can explain to a layman. That's true for most problems I've listed, though more so for this one.

- 2. For which triangles T and numbers k is the following true: for any k-coloring of  $R^2$  there will be a triangle congruent to T such that all three corners have the same color. Other shapes have also been considered. It is known to be true for right triangles and k = 2, but not true for equilateral triangles for any k. For k = 7 no theorem of this type is true since you can ban any particular distance. I've heard of some of these results and I've always meant to learn more. This book gives me an excellent set of references to look up. Though I'll probably read the chapter on Euclidean Ramsey Theory in the Graham-Rothschild-Spencer book first.
- 3. Let  $n, k \in N$ . Take  $K_n$  and draw it in the plane. Then color the edges of it. Are you guaranteed to get a monochromatic non-crossing cycle of length k? It is known that you are not guaranteed  $\sqrt{n} + 1$ . One can look at paths instead of cycles. A Ramsey-type problem that I have never heard of! I'm impressed!

## 2.4 Miscellaneous

- 1. The crossing number of a graph is the min number of crossings you get when you draw it in the plane. The crossing number of  $K_n$  is  $\Theta(n^2)$  and for  $K_{m,m}$  is  $\Theta(n^2m^2)$ . Pin down those contants. I'm surprised that this is open. Live and learn!
- 2. Given n, how many points can you pick from the  $n \times n$  grid such that no three of them are collinear? For  $n = 2, 3, \ldots, 52$  computer seaches have yieled 2n. The best known results is (roughly) 3n/2. The conjecture is 2n. Wow, I never heard of this lovely problem!

# 3 Opinion

This is a very nice collection of problems. If you want to learn some open problems in Discrete Geometry this is clearly the book to go to. You can dip into the book at random and find something of interest. Or you can take some field you've heard of (e.g., in my case Euclidean Ramsey Theory) and use it as a guide to the literature. While the intended audience is Reserachers and Graduate Students, this is a field where people can, with minimal training, understand and enjoy the problems. An interested undergraduate, or someone in a different field, can get a start in this field through this book.