Review of
200 Problems on Language, Automata & Computation
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1 Introduction

The title of the book describes its content so well that I have little to add except this: Since the phrase *Languages, Automata, & Computation* may mean different things to different people I list the chapters:

1. Part I: Problems
   (a) Words, Numbers, Graphs
   (b) Regular Languages
   (c) Context Free Grammars
   (d) Theory of Computation (This chapter includes computational complexity.)
2. Part II: Solutions
   (a) Words, Numbers, Graphs
   (b) Regular Languages
   (c) Context Free Grammars
   (d) Theory of Computation
3. Further Reading
4. Index

2 Summary

The problems each have one of four markings (if you count NO marking as a marking).

1. Easy-*☆
2. Intermediate-unmarked,
3. Hard-$\star$,
4. Very Hard-$\star\star$.

Solutions are included which is either good or bad depending on how you intend to use it. Since some of the problems are hard, I like that there are solutions. The book does not that much on P and NP which is a negative if you are using this for a course that has those topics.

Are these good problems? Absolutely yes. There were many that I have not seen or thought of before, and note that I have taught automata theorem $\aleph_0$ times and TAed 4 times. There is a wide range of problems, and the makings help you tell which is which.

The rest of this review will be a few problems from each chapter, some commentary, and a final opinion. When I give a problem I paraphrase it since there are legal issues with quoting a book verbatim.

2.1 Words, Numbers, Graphs

I expected this chapter to have all easy and intermediary problems. It did not! It has 2 easy, 2 intermediary, 2 hard, and 1 very hard.

$\star\star$ **Definition** The *Thue Morse sequence* is defined as follows: The first symbol is 0. Assume you have the first n symbols t_n. The next n symbols are t'_n which is t_n with the 0's changed to 1's and the 1's changed to 0's. (The problem gives two definitions and asks you to prove they are equivalent.)

1. Show that the Thue-Morse sequence is *cube free*. That is, there is no subword of the form www where $w \in \{0,1\}^+$. (The Thue-Morse sequence is actually *strongly cube free*: there is no subword of the form $bwbwbwb$ where $b \in \{0,1\}$ and $b \in \{0,1\}$.

   They give a hint, but I will not. So perhaps my version is $\star\star\star$.

2. Construct a sequence over a 4-letter alphabet that is square-free. That is, there is no subword of the form ww where $w \in \{0,1\}^+$.

3. Construct a sequence over a 3-letter alphabet that is square-free?

4. Construct a sequence over a 2-letter alphabet that is square-free?

2.2 Regular Languages

1. (no stars nor the empty star) Let $\Sigma = \{0,\ldots,9\}$ and view the input as a number in base 10. Show that

$$\{w \colon w \equiv 0 \pmod 7\}$$

   is regular.

2. $\star$ An *infix* of a word $w = \sigma_1 \cdots \sigma_n$ is any string of the form $\sigma_i \cdots \sigma_j$ where $i \le j$. A language L is *closed under infix* if for all $w \in L$, for all infix's v of w, $v \in L$.

   Give an example of an infinite language that is closed under infix but does not contain an infinite regular language as a subset.

3. $\star\star$ Let $\Sigma = \{a, b\}$. If w is a word and $\sigma \in \{a, b\}$ then let $\#_\sigma(w)$ is the number of σ's in w.

   Give an algorithm for the following: It need not be efficient.

(a) Given a DFA for L determine if, for all $w \in L$, $\#_a(w) = \#_b(w)$.

(b) Given a DFA for L determine if there exists an infinite number of $w \in L$ such that $\#_a(w) = \#_b(w)$.

2.3 Context-Free Languages

1. Give an algorithm that will, given a CFG G, determines if $L(G)$ is infinite.
2. $\star$ Let G be a CFG. Show that the membership question for $L(G)$ is in linear time.
3. $\star$ Show that the set of palindromes cannot be recognized by a deterministic Push Down Automata.

2.4 Theory of Computation

1. $\star$. Let $X \subseteq \mathsf{N}$. Show that X is decidable iff either X is finite or X is the image of a computable strictly increasing function. (I think this question should be 0 stars.)
2. $\star$. Is the following problem decidable: given $u, v \in \Sigma^*$ and a number k is there a string w of length at least k such that $\#_u(w) = \#_v(w)$.
3. Show that the following problem is NP-complete: given a regular expression α over Σ ($|\Sigma|$ might be large) is there a word $w \in L(\alpha)$ such that ever letter in Σ appears in it. (I think this question should be $\star$.)

3 Opinion

I began reading this book skeptical that I would find problems in it that I had not already seen. I was wrong. There are many great problems in this book, some for your students as HW, some to be the basis of lectures you will give your students (I will definitely teach the last problem on Theory of Computation that I stated the next time I teach Automata theorem), and some for your own edification.

The only criticisms are that there is not enough problems on NP-completeness or complexity theory in general.