

1. *Some small cases of  $(a, b, k)$  can likely be worked out explicitly.*

So, it isn't open for all values?

BILL TO DAVID: I am the first one to come up with this open problem so to my knowledge its open. However, for some small values of  $(a, b, k)$  it can probably be worked out. I added:

*Some small cases of  $(a, b, k)$  can likely be worked out explicitly; however, to my knowledge, nobody has done so.*

2. *perhaps impossible in practice, though not in any technical sense*

Don't know what that means.

BILL TO DAVID: I was trying to say that getting lower bounds on  $BB(n)$  may be impossible using the word in English, which means can't be done, but is NOT like saying the HALTING PROBLEM is impossible to solve. The difference between ENGLISH-impossible and MATH-impossible is NOT a point worth getting into. So I deleted part of the sentence.

3. *the fundamental sequence*

Is this notation well known? I don't understand it. Is  $\lambda[n]$  the  $n$ th element in the sequence? If so, what does the  $[n]$  at the end of the first bullet mean? Is it referring to the fundamental sequence for  $\omega^{\alpha_k}$ .

BILL TO DAVID. Yes it is well known. I have added more examples so now you should be able to understand it.

4. *We've met  $e_0$  before, it's  $\epsilon_0$ .*

Is this notation standard.  $e$  and  $\epsilon$  look awfully similar.

BILL TO DAVID: The source I originally used did indeed use  $e_0, e_1, \dots$ . However, this is only one of many standards. Using  $\epsilon_i$  is acceptable and now I do that.

5. *We do not know of any natural function whose growth rate is comparable to*

Who is *We*? Is it you authors or mathematicians?

BILL TO DAVID: I clarified that it is I who does not know of any natural function with these growth rate

6. *I call this representation Veblen Normal Form*

Do you mean this is not standard terminology?

BILL TO DAVID: This is standard and I have made the correction to say so.

7. *well-quasi-order* Suddenly hyphens. Hyphens or no hyphens?

BILL TO DAVID: ALL hypens now, which is standard.

8. Half the time you say well-quasi-ordering, half well-quasi-order. Pick one.

BILL TO DAVID: I asked Google about *linear order* and *linear ordering*:

*"Linear order" and "linear ordering" are largely interchangeable, referring to a total ordering where every element in a set is comparable. In mathematics, "linear order" often emphasizes the structural relationship (the set of pairs), while "linear ordering" often emphasizes the process or the relation itself. Both are used synonymously with total order.*

I was not emphasizing either but I like *well-quasi-ordering* so I changed them all to that.

9. *Let  $x_1$  be that graph* Not to be confused with the  $x_1$  in the beginning of the proof. If only there were more letters that we could use.

BILL TO DAVID: I now use  $y_1$  and  $y_2$  and  $y_i$ .

10. *Take  $n$  to be the least such  $n$*  I'm skimming over the details, so maybe I'm just confused. But, I'm having trouble following the logic. Why does it have to be the least such  $n$ ?

BILL TO DAVID: We do not need to take the least such. For the function that exists because of the theorem, we will take the least  $n$ . I have edited the theorem to say all of that.

11. *Def 8.4 . . .  $f(k)$  is the least  $n$  such that* Seems like this should be on the previous line and preceded by "such that". Ditto for Def 8.6.

BILL TO DAVID: My changes from the last correction made Def 8.4 go away.

12.  *$H$  is a minor of  $G$ .* Not sure what a "minor" is, but maybe this should be "means that  $H$  is a minor of  $G$ ".

BILL TO DAVID. The definition of minor and homeomorphc embedding is now in the paper.

13. [Bru24] The URL is going past the page margin.

BILL TO DAVID: This does not matter. The point of the url is that someone reading the pdf can click on it and get to the papers. That works even if the line goes over the margin.