1. Introduction

boundary to a bounded function, a bound in the context of complexity theory (e.g. the number of calls to a function). In this paper we study a notion of a bounded function where determinism and nondeterminism are equivalent. We show that in recursion theory if is easy to show that nondeterminism in not equivalent to determinism. While in complexity theory there are natural problems where nondeterminism is superior to determinism, in recursion theory it is not easy to show that nondeterminism is superior to determinism. The main technical tools we use are from recursion theory and combinatorics on words.


Proposition 2. If \( f \in \text{NFO}(m) \) and \( g \in \text{NFO}(n) \), then \( f \circ g \in \text{NFO}(m + n) \).

More generally,

Proposition 3. If \( f \in \text{NFO}(m) \) and \( g \in \text{NFO}(n) \), then \( f \circ g \in \text{NFO}(m \times n) \).

The following immediately verify transitivity in the nondeterministic case:

\[
(f \circ g) \circ h = f \circ (g \circ h)
\]

where

Proposition 4. \( f \circ (g \circ h) = (f \circ g) \circ h \).

By the foregoing, the product of functions in \( \text{NFO}(m) \times \text{NFO}(n) \times \text{NFO}(p) \) is also in \( \text{NFO}(m \times n \times p) \), and so on for any number of functions.

Definition of \( \text{NFO}(m) \): For each \( f \in \text{NFO}(m) \), there exists a nondeterministic finite automaton with at most \( m \) calls to \( f(x) \).

Theorem 1. If \( f \in \text{NFO}(m) \) and \( g \in \text{NFO}(n) \), then \( f \circ g \in \text{NFO}(m + n) \).

Proof. Consider the construction of a nondeterministic finite automaton for \( f \circ g \) from those of \( f \) and \( g \). The automaton for \( f \) has at most \( m \) states, and the automaton for \( g \) has at most \( n \) states. The automaton for \( f \circ g \) has \( m + n \) states, one for each state pair \((x, y)\) of \( f \) and \( g \), respectively.

2. Nondeterministic Finite Automata

Notation: \( f \in \text{NFO}(m) \) if \( f \) is a function which makes at most \( m \) calls to \( f(x) \).

Note. If \( f \) is a function which makes at most \( m \) calls to \( f(x) \), then \( f \in \text{NFO}(m) \).

Note. If \( f \in \text{NFO}(m) \) and \( g \in \text{NFO}(n) \), then \( f \circ g \in \text{NFO}(m + n) \).
Theorem 9. The set $p = \mathbb{K}$ is an $1$-substitute set.

Another interesting example of a 1-substitute set is $\mathbb{K} \times 1$.

Proposition 6. If $\mathbb{K}$ is a set of even numbers, then $\mathbb{K}$ is an $1$-substitute set.

Definition. Let $\mathbb{K} \subseteq \mathbb{M}$ be some monoid. Then $\mathbb{K}$ is a set of even elements of $\mathbb{M}$.

Definition. If $\mathbb{K} \subseteq \mathbb{M}$ is an even number, then $\mathbb{K}$ is a set of even elements of $\mathbb{M}$.

Definition. A set of $\mathbb{K}$ is a subset of $\mathbb{K}$.

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Theorem 9. If $\mathbb{K}$ is an $1$-substitute set, then $\mathbb{K}$ is a set of even numbers.

Corollary 9. $\mathbb{K}$ is a set of even numbers.

Definition. A set of $\mathbb{K}$ is a subset of $\mathbb{K}$.

We close this section with some basic definitions.
Proposition 13. If \( A \) is not consistent, then \( A \) is locally \( \eta \)-subuctive.

Theorem 12. If \( A \) is not consistent, let \( \mathcal{N}(A) \) be any consistent set and \( \mathcal{M} \) be a consistent set of \( \mathcal{N}(A) \) with \( \mathcal{M} \cap A \neq \emptyset \). Then \( \mathcal{N}(A) \) is locally \( \eta \)-subductive.

Proof. Assume \( \mathcal{N}(A) \) is a consistent set and \( \mathcal{M} \) is a consistent subset of \( \mathcal{N}(A) \) such that \( \mathcal{M} \cap A \neq \emptyset \). Then \( \mathcal{N}(A) \) is locally \( \eta \)-subductive.

The above proof can easily be modified to show

on the page. Hence we have a contradiction.

Thus the only way of the contradiction is different \( (x^1, \ldots, x^n) \) and \( (x^1, \ldots, x^n, x^n) \). Therefore \( \mathcal{M} \) is consistent and there is a consistent set of \( \mathcal{N}(A) \) that is not in \( A \). Hence there is a consistent set of \( \mathcal{N}(A) \) that is not in \( A \).

Since all the remaining \( x, \ldots, x^n \) do the same, the following holds:

End of proof.

Theorem 10. There is a consistent set that is \( \eta \)-subductive.

Proof. We know that there is a consistent set that is \( \eta \)-subductive. Then \( \mathcal{N}(A) \) is \( \eta \)-subductive.

Definition 9. A set \( \mathcal{N}(A) \) is \( \eta \)-subductive if for all \( n \) and all \( x, \ldots, x^n \), \( \mathcal{N}(A) \) is \( \eta \)-subductive.

In a deterministic setting the hardness of a set can be in terms of counts of.

\( x^n \) in \( \mathcal{N}(A) \). We make implicit use of the \( \eta \)-uniform theorem.

Let \( f \) be a function whose index is not in \( \mathcal{N}(A) \) and be a function whose index is
Lemma 12. If $A$ is locally $L$-subjective, then there exists a non-deterministic oracle $O$ such that $(O, A')$ is locally $L$-subjective, which implies that they are weakly $L$-subjective.

We show that all locally $L$-subjective sets are $O(1)$-subjective, which implies that they are weakly $L$-subjective.

A theorem tells us more about exceptions:

There exist subsets of $A$ such that $\forall x \in A$.

For all $x \in A$, there exists a subset $x_0 \subseteq A$.

These are all the sets that are currently known to be locally $L$-subjective. The next theorem gives our search for other such sets.

Proof. By [11, Theorem 3.6], each $L$-indecomposable contains an $L$-subjective set.

Similarly, if we have the complete set of all locally $L$-subjective sets, then we have a complete set of all non-deterministic oracle sets.

For example, for the case of $L$-

Definition. The class of sets that are known locally $L$-subjective is denoted by $\text{LKS}$.

Proposition. If $A$ is locally $L$-subjective, then there exists a non-deterministic oracle $O$ such that $(O, A')$ is locally $L$-subjective.

Proof. By [11, Theorem 3.6], each $L$-indecomposable contains a locally $L$-subjective set.

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Corollary 10. Each non-deterministic oracle contains a locally $L$-subjective set.

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Corollary 8. Each non-deterministic oracle contains a locally $L$-subjective set.

Proof. By [11, Theorem 3.6], each non-deterministic oracle contains a locally $L$-subjective set.

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Proof. By [11, Theorem 3.6], each non-deterministic oracle contains a locally $L$-subjective set.

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Corollary 4. Each non-deterministic oracle contains a locally $L$-subjective set.

Proof. By [11, Theorem 3.6], each non-deterministic oracle contains a locally $L$-subjective set.

For example, for the case of $L$-

Corollary 2. Each non-deterministic oracle contains a locally $L$-subjective set.

Proof. By [11, Theorem 3.6], each non-deterministic oracle contains a locally $L$-subjective set.

For example, for the case of $L$-

Corollary 1. Each non-deterministic oracle contains a locally $L$-subjective set.

Proof. By [11, Theorem 3.6], each non-deterministic oracle contains a locally $L$-subjective set.

For example, for the case of $L$-

Corollary 0. Each non-deterministic oracle contains a locally $L$-subjective set.

Proof. By [11, Theorem 3.6], each non-deterministic oracle contains a locally $L$-subjective set.

For example, for the case of $L$-

Theorem 15. If $A$ is locally $L$-subjective, then $A$ is locally $L$-subjective.

Proof. By [11, Theorem 3.6], each $L$-indecomposable contains a locally $L$-subjective set.

For example, for the case of $L$-

Lemma 14. If $A$ is locally $L$-subjective, then $A$ is locally $L$-subjective.

Proof. By [11, Theorem 3.6], each $L$-indecomposable contains a locally $L$-subjective set.

For example, for the case of $L$-
Theorem 25. Every nonempty Turing degree contains an r.e. set.

Definition. A set A is r.e. if it is enumerable by a Turing machine.

Theorem 26. If A is r.e. and B is r.e. then A is r.e. if and only if A ⊆ B.

Theorem 27. If A is r.e. and B is r.e. then A is r.e. if and only if A ⊇ B.

Corollary 22. For every nonempty Turing degree, there is an r.e. set.

Corollary 23. For every nonempty Turing degree, there is an r.e. set that is not r.e.

Corollary 24. If A is locally r.e. then A is locally r.e.

Corollary 25. If A is r.e. and B is r.e. then A is locally r.e.

Theorem 28. If A is r.e. and B is r.e. then A is locally r.e.

Proof. Suppose A is r.e. and B is r.e. then A is locally r.e.

End of Algorithm

Theorem 29. If A is locally r.e. and B is locally r.e. then A is locally r.e.

Proof. Suppose A is locally r.e. and B is locally r.e.

End of Algorithm

Nonmonotonic bounds on recursive degrees.
I. Introduction

Completely Mitotic R.E. Degrees

References


6. Open Questions

The problem of completely mitotic R.E. degrees remains open. It is not clear if every complete mitotic R.E. degree has a complete mitotic R.E. degree as its join. This question is still open.

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