Handed out September 9, 2011. Due within the first 15 minutes of class September 22, 2011.

Note: You may discuss homework problems and general solution strategies with classmates. However, you must write up the solutions yourself, and document any and all resources appropriately.[1]

Problem 1. Derive a closed form solution for the following summation using constructive induction assuming that $n \geq 0$.

$$\sum_{i=0}^{n} i^3$$

Problem 2. Prove by mathematical induction on n, where $n \geq 0$:

$$\sum_{i=0}^{n} (i+1)\ i\ (i-1) = \left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)}{2}$$

Problem 3. The product of two $n \times n$ matrices $A * B$ is defined to be the $n \times n$ matrix C where

$$c_{ij} = \sum_{k=0}^{n-1} a_{ik} b_{kj}.$$

- **a.** Write pseudo code to multiply two nxn matices.
- **b.** Use a triple sum to count the number of multiplications.
- **c.** Simplify the summation.
- **d.** Use a triple sum to count the number of additions. Simplify the summation.

Problem 4 Consider a slightly different matrix multiplcation problem. Assume that U is an $n \times n$ upper triangular matrix, where the entries below the diagonal are always zero. That is, for $i > j$, the value of $u_{ij} \equiv 0$. Assume that L is an $n \times n$ lower triangular matrix, in which the entries above the diagonal are all zero. That is, for $i < j$, the value of $l_{ij} \equiv 0$.

- **a.** Write psuedo code to (efficiently) multiply an nxn upper triangular matrix, U, by an nxn lower triangular matrix, L.
- **b.** Use a triple sum to count the number of multiplications.
- **c.** Simplify the summation.
- **d.** Use a triple sum to count the number of additions. Simplify the summation.

[1] If I were documenting my lectures or this homework, I'd have to write something like: Katz, Mitchell, private conversation. Kruskal, Clyde, inspired problem 3 and 4.